Style Investing

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Abstract

We study asset prices in an economy where some investors categorize risky assets into different styles and move funds among these styles depending on their relative performance. Our model implies that assets in the same style comove too much while assets in different styles comove too little, and that high average returns on a style are associated with common factors for reasons unrelated to risk. Our model also generates a rich pattern of own- and cross-autocorrelations, and implies that style-level momentum and value strategies can be as profitable as their individual stock-level counterparts. Finally, the model sheds light on anomalies related to some special styles, such as indexation.

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1 Introduction

One of the clearest mechanisms of human thought is classification: the grouping of objects into categories based on some similarity among them (Rosch and Lloyd 1978, Wilson and Keil 1999). We group countries into democracies and dictatorships based on features of political systems within each group. We classify occupations as blue collar or white collar based on whether people work primarily with their hands or with their heads. We put foods into categories such as proteins and carbohydrates based on their nutritional characteristics.

Classification of large numbers of objects into categories is also pervasive in financial markets. When making portfolio allocation decisions, many investors first categorize assets into broad classes such as large cap stocks, value stocks, government bonds or venture capital, and then decide how to allocate their funds across these various asset classes (Bernstein 1985, Swensen 2000). The asset classes that investors use in this process are sometimes called "styles," and the process itself, namely allocating funds among styles rather than among individual securities, is known as "style investing." In this paper, we analyze financial markets in which many investors pursue style investing.

Assets in a style or class typically share a common characteristic, which can be based in law (e.g., government bonds), in markets (e.g., small cap stocks), or in fundamentals (e.g., real estate). In some cases, the cash flows of assets in the same style are highly correlated, as with automotive industry stocks, while in other cases, such as closed-end funds, they are largely uncorrelated. Some styles are relatively permanent over the years (e.g., U.S. government bonds), while others come (e.g., small stocks) and go (e.g., railroad bonds). One reason for the appearance of a new style is financial innovation, as when mortgage-backed securities were invented. Another reason is the detection of superior performance in a group of securities with a common characteristic: small stocks became an investment style following Banz' (1979) discovery of the small firm effect. Styles typically disappear after long periods of poor performance, as was the case with railroad bonds.

There are several reasons why both institutional and individual investors might pursue style investing. First, categorization simplifies problems of choice and allows us to process vast amounts of information reasonably efficiently (Mullainathan 2000). Allocating money across ten asset styles is far less intimidating than choosing among the thousands of listed securities. Second, the creation of asset categories helps investors evaluate the performance of professional money managers, since a style automatically creates a peer group of managers who pursue that particular style (Sharpe 1992). Money managers are now increasingly evaluated relative to a performance benchmark specific to their style, such as a growth or a value index. Third, style investing simplifies the process of diversification: instead of checking for over-exposure to an economic factor by studying the characteristics of individual assets, the investor can simply examine the properties of styles.[1]

These benefits of style investing are particularly attractive to institutional investors, such as pension plan sponsors, foundations and endowments, who as fiduciaries must follow systematic rules of portfolio allocation. Perhaps for this reason, interest in style investing has grown over the years, paralleling the growth of institutional investors. Not surprisingly, the financial services industry has responded to this interest. Most pension fund managers, as well as some mutual fund managers catering to the needs of individual investors, now identify themselves as following a particular investment style, such as growth, value, or technology.[2]

The growing importance of style investing points to the usefulness of assessing its effect on financial markets and security valuation. In this paper, we present a simple model that allows for such an assessment. The model combines style-based portfolio selection strategies of investors with a plausible mechanism for how these investors choose among styles. Specifically, we assume that many investors move into styles that have performed well in the past, financing this shift by withdrawing funds from styles that have performed poorly, and also that these fund flows affect prices. We think of these investors' policy as arising from a combination of trend-chasing and a reluctance to let their allocation to the broadest asset classes – cash, bonds, and stocks – drift too far from a preset target level.

The simple assumptions in our model generate a number of empirical predictions, some already available in the theoretical literature, others entirely new. We find that grouping assets into the same style leads to comovement in their returns, even when their fundamentals are unrelated. In other words, style investing by itself induces common factors in security returns. These factors are in some instances accompanied by higher *average* returns for reasons that have nothing to do with risk. We also find that when an asset is reclassified into a new style, it comoves more with that style after reclassification than before. Moreover, while assets in the same style comove more than their fundamentals, assets in different styles comove *less* than their underlying cash flows.

Our model predicts a rich structure of return autocorrelations. Some of these predictions – in particular, those about own-autocorrelations – are shared with earlier models. Other predictions, about cross-autocorrelations of returns across investment styles, are more unique to our framework. We also show that in our economy, individual stock-level momentum and value strategies are profitable. While this effect can be found in earlier work, our model

[1] Style investing is not without cost. In treating all assets within a class as being alike, categorization entails a loss of information that is possibly relevant to portfolio allocation. This cost is likely to be more important when the cash-flow fundamentals of the assets in a style are largely unrelated.

[2] As indicated earlier, we use the term "style investor" to refer to investors such as pension plan sponsors who allocate funds at the style level, rather than at the individual asset level. The term can also be used in a related, but distinct sense to describe money managers who restrict themselves to picking stocks from within a specific asset style. While both uses of the term are common in practice, in this paper "style investor" refers only to the investors providing the funds, and not to the money managers they hire.

yields the additional prediction that style-level momentum and value strategies can be as profitable or even more profitable than their individual stock-level counterparts.

In essence, our predictions about time series autocorrelations reflect the fact that in our model, investment styles follow a specific life cycle. The birth of a style is often triggered by good fundamental news about the securities in a style. The style then matures as its good performance recruits new funds, further raising the prices of securities belonging to the style. Finally, the style collapses, either because of arbitrage or because of bad fundamental news. Over time, the style may be reborn.

We use our model to shed light on a number of puzzling empirical facts. Among other phenomena, we address the excess comovement of small stocks and value stocks (Fama and French 1995), the growing importance of industry factors in European equity returns, the performance of the small stock investment style over time, and the poor performance of value stocks during 1998-1999 despite good earnings (Chan, Karceski and Lakonishok, 2001).

Of the three elements of our model – investors' policy of allocating funds at the style level, their trend-chasing, and their reluctance to let their allocation to the broadest asset classes drift away from a target level – only one, trend-chasing, has received much prior attention in the theoretical literature: De Long, Shleifer, Summers and Waldmann (1990a) and Hong and Stein (1999) both consider models with trend-chasing investors. Neither of these papers studies the effect of classifying assets into styles, nor of having target allocations for broad asset classes. In part this is because earlier research has typically worked with only a single asset, while style investing pertains to multi-asset economies.

In Section 2, we construct a simple model of style investing. Section 3 develops some of the intuition that lies behind the model's predictions. In Section 4, we lay out the model's implications in a series of formal propositions. Section 5 analyzes two specific kinds of styles – indices and price-dependent styles – in more detail, while Section 6 concludes.

2 A Model of Style Investing

2.1 Assets and Styles

We consider an economy with $2n$ risky assets in fixed supply, and a riskfree asset, cash, in perfectly elastic supply and with zero net return. Following Hong and Stein (1999), we model risky asset i as a claim to a single liquidating dividend $D_{i,T}$ to be paid at some later time T. The eventual dividend equals

$$D_{i,T} = D_{i,0} + \varepsilon_{i,1} + \ldots + \varepsilon_{i,T}, \tag{1}$$

where $\varepsilon_{i,t}$ becomes known at time t. We assume

$$\varepsilon_t = (\varepsilon_{1,t}, \ldots, \varepsilon_{2n,t})' \sim N(0, \Sigma_D), \text{ i.i.d over time.}$$

The price of a share of risky asset l at time t is $P_{l,t}$ and the return on the asset between time $t-1$ and time t is[3]

$$\Delta P_{l,t} = P_{l,t} - P_{l,t-1}. \quad (2)$$

We assume that to simplify their decision-making, some investors in the economy group the risky assets into a small number of categories, which we refer to as styles, and express their demand for risky assets at the level of these styles. In other words, a style is a group of risky assets that some investors do not distinguish between when formulating their demand.

To test any predictions that emerge from a style-based model, it is important to have a concrete way of identifying styles. One way of doing this is to look at the labels that mutual and pension fund managers use to describe their products to clients. If money managers are responsive to client needs, they will choose labels that correspond to the categories people like to use when thinking about investments. For example, since many money managers offer funds that invest in small stocks, "small stocks" may be a style in the minds of many investors. Large stocks, value stocks, growth stocks, and stocks within a particular industry, country, or index are then also all examples of styles.

The styles that investors use in financial markets support the idea that when forming categories, people tend to group together objects that are similar in having a common characteristic. Small stocks, for example, share the characteristic that they have low market capitalizations. Not every group of stocks with a common characteristic becomes an investment style. There is no evidence that investors group stocks that begin with the letter "A" into a style; certainly, money managers do not offer any investment products that facilitate this kind of thinking.

We build a simple model of style investing. There are just two styles, X and Y, and each risky asset in the economy belongs to one, and only one, of these two styles. Risky assets 1 through n are in style X while $n+1$ through $2n$ are in style Y. For now, we assume that this classification is permanent: the composition of the two styles is the same in every time period. It may be helpful to think of X and Y as "old economy" stocks and "new economy" stocks, say.[4]

As a measure of the value of style X at time t, we use $P_{X,t}$, defined as the average price

[3]For simplicity, we abuse terminology slightly and refer to the asset's change in price as its return.

[4]More generally, a given security may belong to multiple overlapping styles. A small bank stock with a low price-earnings ratio may be part of a small stock style, a financial industry style, and a value style. A model capturing such overlaps can be constructed and would yield similar but less transparent predictions.

of a share across all assets in style X:

$$P_{X,t} = \frac{1}{n} \sum_{l \epsilon X} P_{l,t}. \tag{3}$$

The return on style X between time $t-1$ and time t is

$$\Delta P_{X,t} = P_{X,t} - P_{X,t-1}. \tag{4}$$

Although our model does not require it, we restrict attention to simple cash-flow covariance structures. In particular, we suppose that the cash-flow shock to an asset has three components: a market-wide cash-flow factor which affects assets in both styles, a style-specific cash-flow factor which affects assets in one style but not the other, and a completely idiosyncratic cash-flow shock specific to a single asset. Formally, for $i \epsilon X$,

$$\varepsilon_{i,t} = \psi_M f_{M,t} + \psi_S f_{X,t} + \sqrt{(1 - \psi_M^2 - \psi_S^2)} f_{i,t},$$

and for $j \epsilon Y$,

$$\varepsilon_{j,t} = \psi_M f_{M,t} + \psi_S f_{Y,t} + \sqrt{(1 - \psi_M^2 - \psi_S^2)} f_{j,t},$$

where $f_{M,t}$ is the market-wide factor, $f_{X,t}$ and $f_{Y,t}$ are the style-specific factors, and $f_{i,t}$ and $f_{j,t}$ are the idiosyncratic factors; ψ_M and ψ_S are constants which control the relative importance of the three components. We suppose that each factor has the same unit variance and is orthogonal to the other factors. This implies

$$\Sigma_D^{ij} \equiv \text{cov}(\varepsilon_{i,t+1}, \varepsilon_{j,t+1}) = \begin{cases} 1, \; i = j \\ \psi_M^2 + \psi_S^2, \; i \neq j, \; i,j \text{ in the same style} \\ \psi_M^2, \; i \neq j, \; i,j \text{ in different styles.} \end{cases} \tag{5}$$

In words, all assets have a cash-flow news variance of one, the pairwise cash-flow correlation between any two assets in the same style is the same, and the pairwise cash-flow correlation between any two assets in different styles is also the same.

2.2 Switchers

There are two kinds of investors in our model, "switchers" and "fundamental traders." The investment policy of switchers has two distinctive characteristics. First, they allocate funds at the level of a style. Second, how much they allocate to each style depends on that style's past performance *relative to other styles*. In other words, each period, switchers allocate more funds to styles with better than average performance and finance these additional investments by taking funds away from styles with below average performance. To capture this, we write their demand for shares of an asset i in style X at time t as

$$N_{i,t}^S \equiv \frac{N_{X,t}^S}{n} \equiv \frac{1}{n} \left[A_X + \sum_{k=1}^{t-1} \theta^{k-1} \left(\frac{\Delta P_{X,t-k} - \Delta P_{Y,t-k}}{2} \right) \right], \tag{6}$$

where A_X and θ are constants, with $0 < \theta < 1$. Symmetrically, switcher demand for shares of an asset j in style Y at time t is

$$N_{j,t}^S \equiv \frac{N_{Y,t}^S}{n} \equiv \frac{1}{n}\left[A_Y + \sum_{k=1}^{t-1} \theta^{k-1}\left(\frac{\Delta P_{Y,t-k} - \Delta P_{X,t-k}}{2}\right)\right]. \quad (7)$$

In words, when deciding on their time t allocation, switchers compare style X's and style Y's return between time $t-2$ and time $t-1$, between time $t-3$ and time $t-2$, and so on, with the most recent past being given the most weight. They then move funds into the style with the better prior record, buying an equal number of shares of each asset in that style, and reduce their holdings of the other style. The parameter θ determines how far back switchers look when comparing the past performance of styles, and hence, the persistence of their flows. A_X can be thought of as switchers' long run target demand for style X, from which they deviate based on the relative performance of styles.[5]

The fact that switcher demand for all assets within a given style is the same underscores our assumption that switchers allocate funds at the style level and do not distinguish among assets in the same style. Of course, since assets in the same style have a common cash-flow factor, it is not unreasonable for investors to view them as similar. In allocating according to (6) and (7), however, investors treat assets as more similar than they really are.

We think of the relative performance feature in (6) and (7) as arising from a combination of trend-chasing and a reluctance on the part of investors to let their overall allocation to the broadest asset classes – cash, bonds, and stocks – drift too far from a preset target level; institutional investors in particular try to keep their allocations to these three asset classes close to predetermined targets.[6] The intuition for how this combination leads to the allocations in (6) and (7) is straightforward. Holding everything else constant, an increase in $\Delta P_{X,t-1}$, the most recent past return for old economy stocks, leads switchers to forecast higher returns on that style in the future, and hence to increase their demand for it at time t. However, since they want to keep their overall allocation to equities unchanged, they have to sell shares of new economy stocks, style Y. Therefore, $\Delta P_{X,t-1}$ has an equal and opposite effect on $N_{i,t}^S$ in (6) and $N_{j,t}^S$ in (7), making demand a function of relative past performance.[7]

Trend-chasing can itself be motivated in a number of ways. It can be the result of a cognitive bias which leads investors to put more weight on past returns than they should when

[5]The strategies in (6) and (7) are not self-financing. Rather, we assume that at the start of each period, switchers are endowed with sufficient resources to fund their strategies. This allows us to abstract from issues which are not our main focus here – the long run survival of switchers, for example – and to concentrate on understanding the behavior of prices when switchers play a role in setting them.

[6]See Swensen (2000) for more discussion of institutional investor behavior.

[7]The Appendix to Barberis and Shleifer (2000) shows more formally how trend-chasing combined with a constraint on asset class allocations leads to demand functions like those in (6) and (7).

forecasting future returns.[8] It can also stem from agency considerations: an institutional investor, such as the sponsor of a defined benefit plan, may hire money managers with strong prior records and fire those with poor performance simply because such strategies are easier to justify ex-post to those monitoring their actions.

Trend-chasing also emerges as the reduced form policy in an important class of models in which investors are uncertain about the growth rate of an asset's cash flows and are forced to learn it by observing cash-flow realizations; after several periods of impressive cash-flow growth, for example, they raise their estimate of the rate (Brennan and Xia 1999, Veronesi 1999, Lewellen and Shanken 2002). Our model can then be interpreted as a conventional learning model coupled with two less conventional assumptions: that investors learn at the style level, perhaps again to simplify their decision-making, and that they stick closely to target allocations for the broadest asset classes. One difficulty with this interpretation, however, is that it suggests that investors should choose styles with strong cash-flow factors so that their learning is as efficient as possible, while in practice, some styles – small stocks and value stocks – are only weakly associated with cash-flow factors.

Whatever the source, empirical evidence in support of trend-chasing is strong. Patel, Zeckhauser, and Hendricks (1990), Ippolito (1992), Goetzmann and Peles (1997), Chevalier and Ellison (1997), and Sirri and Tufano (1998) show that flows into mutual funds are heavily influenced by past returns; Lakonishok, Shleifer, and Vishny (1992) find that pension fund managers who post high returns subsequently attract significant flows of funds; Choe, Kho, and Stulz (1999) and Froot, O'Connell and Seasholes (2001) show that foreign investors tend to buy into countries with good recent stock market performance.

In reality, investors allocate funds across many different styles, not just two. Even with many styles, though, the two-style formulation in (6) is still relevant, and this is important for the economic significance of our predictions. When an investor pours money into a style he deems attractive, he may well finance this by withdrawing funds from just one other style, rather than from many others. One reason for this is transaction costs: in terms of withdrawal fees and time spent, it is likely to be less costly to take $10 million away from one money manager than to take $1 million away from ten of them.

Another, potentially more important, reason is that there is often a natural candidate style to withdraw funds from, namely a style's *twin style*. Many styles come in natural pairs; stocks with a high value of some characteristic constitute one style, and stocks with a

[8]For example, people often estimate the probability that a data set is generated by a certain model based on the degree to which the data is *representative* or reflects the essential characteristics of the model (Tversky and Kahneman, 1974). A style that has had several periods of high returns is representative of a style with a high true mean return, which may explain why impressive past returns raise some investors' forecasts of future returns. It should be noted that some experiments suggest that people sometimes put too *little* weight on recent information, a phenomenon known as conservatism. However, this typically only arises when subjects have strong prior beliefs in advance of seeing the new data.

low value of the same characteristic, the twin: value stocks and growth stocks are a simple example. When an investor moves into the growth style, the value style is a tempting source of funds. First, because of the way twins are defined, there is no overlap between them. Second, it is easy to succumb to the mistaken belief that since a style and its twin and defined as opposites, their returns will also be "opposite"; if prospects for the growth style are good, prospects for the value style must be bad.

2.3 Fundamental Traders

The second investor type in our model is *fundamental traders*. They act as arbitrageurs, and try to prevent the price of an asset from deviating too far from its expected final dividend. To formalize this, suppose that at the start of each period, fundamental traders are given an amount W^F to allocate. We assume that they have CARA preferences defined over the value of their invested funds one period later, and take price changes to be normally distributed. Since they have no constraints on their allocations, they solve

$$\max_{N_t} E_t^F(-\exp[-\gamma(W^F + N_t{}'(P_{t+1} - P_t))]), \tag{8}$$

where $N_t = (N_{1,t}, \ldots, N_{2n,t})'$ is a vector of the number of shares allocated to each risky asset, γ governs the degree of risk aversion, E_t^F denotes fundamental trader expectations at time t, and $P_t = (P_{1,t}, \ldots, P_{2n,t})'$.

Optimal holdings N_t^F are given by

$$N_t^F = \frac{(V_t^F)^{-1}}{\gamma}(E_t^F(P_{t+1}) - P_t), \tag{9}$$

where

$$V_t^F \equiv \mathrm{var}_t^F(P_{t+1} - P_t),$$

with the F superscript in V_t^F again denoting a forecast made by fundamental traders.

2.4 Pricing

Given fundamental trader expectations about future prices, which we discuss shortly, prices are set as follows. The fundamental traders double up as market makers and treat the demand from switchers as a supply shock. If the total supply of the $2n$ assets is given by the vector Q, equation (9) implies

$$P_t = E_t^F(P_{t+1}) - \gamma V_t^F(Q - N_t^S), \tag{10}$$

where $N_t^S = (N_{1,t}^S, \ldots, N_{2n,t}^S)'$. In contrast to switchers, who form expectations of future prices based on past prices, fundamental traders are forward looking and base price forecasts on expectations about the final dividend. One way they may do this is to roll equation (10) forward iteratively, setting

$$E_{T-1}^F(P_T) = E_{T-1}^F(D_T) = D_{T-1},$$

where $D_t = (D_{1,t}, \ldots, D_{2n,t})'$. This leads to

$$P_t = D_t - \gamma V_t^F(Q - N_t^S) - E_t^F \sum_{j=1}^{T-t-1} \gamma V_{t+j}^F(Q - N_{t+j}^S). \tag{11}$$

Suppose that fundamental traders set

$$V_t^F = V, \forall t, \tag{12}$$

where V has the same structure as the cash-flow covariance matrix Σ_D, so that

$$V^{ij} \equiv \text{cov}(\Delta P_{i,t+1}, \Delta P_{j,t+1}) = \begin{cases} \sigma^2, \; i = j \\ \sigma^2 \rho_1, \; i \neq j, \; i,j \text{ in the same style} \\ \sigma^2 \rho_2, \; i \neq j, \; i,j \text{ in different styles} \end{cases} \tag{13}$$

and also that

$$E_t^F(N_{t+j}^S) = \overline{N}^S. \tag{14}$$

In other words, while fundamental traders recognize the existence of a supply shock due to switchers, they are not sophisticated enough to figure out its time series properties. We return to this assumption later in this section.

Our assumptions imply

$$P_t = D_t - \gamma V(Q - N_t^S) - (T - t - 1)\gamma V(Q - \overline{N}^S). \tag{15}$$

Dropping the non-stochastic terms, we obtain

$$P_t = D_t + \gamma V N_t^S. \tag{16}$$

For the particular form for V conjectured by fundamental traders, this simplifies further. Up to a constant, the price of an asset i in style X is

$$\begin{aligned} P_{i,t} &= D_{i,t} + \gamma\sigma^2(1 - \rho_1 + n(\rho_1 - \rho_2))\frac{N_{X,t}^S}{n} \\ &= D_{i,t} + \frac{1}{\phi}\sum_{k=1}^{t-1} \theta^{k-1}\left(\frac{\Delta P_{X,t-k} - \Delta P_{Y,t-k}}{2}\right), \end{aligned} \tag{17}$$

where

$$\phi = \frac{n}{\gamma\sigma^2(1-\rho_1+n(\rho_1-\rho_2))}, \tag{18}$$

and the price of an asset j in style Y is

$$P_{j,t} = D_{j,t} + \frac{1}{\phi}\sum_{k=1}^{t-1}\theta^{k-1}\left(\frac{\Delta P_{Y,t-k}-\Delta P_{X,t-k}}{2}\right). \tag{19}$$

We study equilibria in which fundamental traders' choices in (12) and (14) are reasonable, in that they lead, through (17) and (19), to prices for which the unconditional covariance matrix of returns equals V, and the unconditional mean of switcher demand actually is $\overline{N}^S$. Such equilibria exist for a wide range of values of the exogeneous parameters Σ_D, A_X, A_Y, θ, and γ.

Note that in a world with only fundamental traders,

$$P_t = D_t. \tag{20}$$

We refer to this as the fundamental value of the assets and denote it P_t^*.

Equation (16) shows that fundamental traders are not able to push prices back to fundamental values. The fact that they have a short one-period horizon forces them to worry about shifts in switcher sentiment and makes them less aggressive in combating any mispricing, a mechanism originally suggested by De Long et al. (1990b). One justification for giving arbitrageurs short horizons, proposed by Shleifer and Vishny (1997), is that if investors are not sophisticated enough to understand their money manager's strategies, they may simply use short-term returns as a way of judging his competence, and withdraw funds after poor performance. The threat of this happening forces arbitrageurs to take a short-term view.

Fundamental traders' inability to wipe out the influence of noise traders is consistent with the substantial body of empirical evidence indicating that uninformed demand shocks influence security prices (Harris and Gurel 1986, Shleifer 1986, Froot and Dabora 1999, Kaul, Mehrotra, and Morck 2000, and Lamont and Thaler 2000). Moreover, if we think of switchers as institutions chasing the best-performing style, our model is consistent with the evidence that demand shifts by institutions in particular influence security prices (Gompers and Metrick 2001).

Even if we were to include more sophisticated arbitrageurs in our model – arbitrageurs who understand the form of demand function (6) – they might not counteract the mispricing to any greater degree; on the contrary, they might exacerbate it. This is the finding of De Long et al. (1990a), who consider an economy with positive feedback traders similar to our switchers, as well as arbitrageurs. When an asset's price rises above fundamental value, the arbitrageurs do not sell or short the asset. Rather, they *buy* it, knowing that the price rise will attract more feedback traders next period, leading to still higher prices, at which point the arbitrageurs can exit at a profit. Since sophisticated arbitrageurs may amplify rather than counteract the effect of switchers, we exclude them from our simple model.

2.5 Parameter Values

In Section 4, we prove some general propositions about the behavior of asset prices in our economy. To illustrate some of these propositions, we use a numerical implementation of (17) and (19) in which the exogeneous parameters Σ_D, A_X, A_Y, θ and γ are assigned specific values.

From (5), the cash-flow covariance matrix is completely determined by ψ_M and ψ_S. We set $\psi_M = 0.25$, and $\psi_S = 0.5$. In this case, the cash-flow covariance matrix is given by

$$\Sigma_D = \begin{pmatrix} A & B \\ B & A \end{pmatrix}, \tag{21}$$

with

$$A = \begin{pmatrix} 1 & 0.31 & \cdots & 0.31 \\ 0.31 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0.31 \\ 0.31 & \cdots & 0.31 & 1 \end{pmatrix}, B = \begin{pmatrix} 0.06 & \cdots & \cdots & 0.06 \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ 0.06 & \cdots & \cdots & 0.06 \end{pmatrix}.$$

The remaining parameters are set equal to

$$\begin{aligned} A_X &= A_Y = 0, \\ \theta &= 0.95, \\ \gamma &= 1.1. \end{aligned} \tag{22}$$

Equation (6) shows that θ controls the persistence of switcher flows; we choose θ close to 1 so as to capture the kind of persistence in fund flows we think is important in financial markets. Fundamental trader risk aversion γ is set so that in equilibrium, returns exhibit a level of excess volatility that is reasonable given historical U.S. data. For these parameter values, there is an equilibrium in which style returns have a standard deviation 1.3 times the standard deviation of cash-flow shocks.[9] In this equilibrium, the value of ϕ in (18) is 1.25.[10]

[9]For comparison, the standard deviation of aggregate dividend growth over 1926-1995 is around 12% while the standard deviation of aggregate stock returns over the same period is around 18%, 1.5 times higher.

[10]The parameter values in (21) and (22) also support other equilibria, including one where returns are only slightly more volatile than cash flows. The intuition is that if fundamental traders *think* that returns are not very volatile, they will trade against switchers more aggressively, with the result that equilibrium returns will indeed have low volatility. To support the equilibrium described in the main text, we need fundamental traders to *expect* returns to be substantially more volatile than cash flows. Reassuringly, all the results in this paper remain qualitatively valid across multiple equilibria.

3 Competition Among Styles

3.1 Impulse Response Functions

As a first step towards understanding the effect of switchers on asset prices, we use the formula for price in equation (17) to generate some impulse response functions. We take $n = 50$, so that there are 100 stocks, the first 50 of which are in style X and the last 50 in style Y. The parameters are set equal to the values in (21) and (22). Figure 1 shows how the prices $P_{X,t}$ and $P_{Y,t}$ of styles X and Y, defined in (3), evolve after a one-time cash-flow shock to style X when $t = 1$. In the notation of our model,

$$\begin{aligned} \varepsilon_{i,1} &= 1, \varepsilon_{i,t} = 0, \ t > 1, \ \forall i \epsilon X, \\ \varepsilon_{j,t} &= 0, \ \forall t, \ \forall j \epsilon Y. \end{aligned}$$

The initial expected cash flows are $D_{i,0} = 50$, $\forall i$.

The solid line in the top half of the graph tracks $P_{X,t}$, the value of style X in the presence of switchers. The dashed line in the top half is the fundamental value of style X, $P^*_{X,t}$, defined through equations (20) and (3) as the value of style X when there are only fundamental traders in the economy and no switchers.

Figure 1 shows that in the presence of switchers, a cash-flow shock to style X leads to a substantial and long-lived deviation of X's price from its fundamental value. The good cash-flow news about X pushes up its price. This outperformance catches the attention of switchers, who then increase their demand for X in the following period, pushing X's price still higher and drawing in more switchers. In the absence of any more good cash-flow news, switchers' interest in style X eventually fades and prices return to fundamental value.

The fact that switchers' investment decisions are based on *relative* rather than absolute past performance leads to a novel prediction which we refer to as an *externality*. Figure 1 shows that the cash-flow shock to X affects not only X's price, but also Y's, even though there has been no news about Y. The good news about X draws funds into that style; however, since switchers want to maintain a constant overall allocation to equities, they finance the extra investment in X by taking money out of Y. This pushes Y's price down, making it look even worse relative to X and leading to still more redemptions by switchers.

In practice, the quantitative magnitude of the externality depends heavily on how investors finance their style shifts. If they finance a shift into a particular style by withdrawing small amounts of money from all other styles, the externality will be dispersed, making it hard to detect. However, as we argue in Section 2, it may be more plausible that investors finance shifts into a style by withdrawing funds from the style's twin alone. When the externality is concentrated in the twin style, it is more easily detectable.[11]

We can also look at impulse responses to *stock-specific* cash-flow news. Suppose that stock 1, a member of style X, experiences a one-time cash-flow shock at time 1. In our notation,

$$\begin{aligned} \varepsilon_{i,1} &= 1, \varepsilon_{i,t} = 0,\ t > 1,\ \text{for } i = 1, \\ \varepsilon_{i,t} &= 0,\ \forall t,\ \text{for } i = 2, \ldots, n. \end{aligned}$$

Figure 2 plots prices $P_{i,t}$ and fundamental values $P^*_{i,t}$ for $i = 1$, 2, and 100. Recall that stocks 1 and 2 are in style X while stock 100 is in style Y.

Figure 2 helps bring out the differences between our model and the related positive feedback trading models of De Long et al. (1990a) and Hong and Stein (1999), in which the feedback occurs at the level of an individual asset, so that noise traders increase their demand for an asset if it had a good return in the previous period. In these earlier models, a cash-flow shock to stock 1 would only push stock 1's price away from fundamental value: stock 1's outperformance would attract the attention of positive feedback traders who would then buy the stock in the next period, pushing its price up too high; stocks 2 and 100, on the other hand, would remain untouched.

Figure 2 shows that in our model, *all three* stocks deviate from fundamental value after the initial cash-flow shock to stock 1. The fact that in our model, stocks 2 and 100 – which received no cash-flow news at all – also move away from fundamental value is due to the two new features of our model: a demand function that is defined at the style level and a focus on relative, rather than absolute performance. The time 1 cash-flow shock to stock 1 boosts not only stock 1's return but also style X's return, attracting attention from switchers, who then allocate more funds to style X at time 2, pushing both stocks 1 and 2 away from fundamental value. Since the inflows to X are financed by withdrawing funds from Y, the price of stock 100 is pushed below fundamental value.

3.2 Discussion

The impulse response functions show that, in our model, styles go through cycles. A style X is set in motion by good fundamental news about itself or alternatively by bad news about *another* style Y, which affects it through the externality. The style then swings away from fundamental value for a prolonged period, powered by fund flows attracted by its superior

[11] Another kind of externality arises when a sector experiences a positive shock to investment opportunities, drawing in capital and driving up interest rates, which then pushes risky asset prices in other sectors down. Our model makes the distinct prediction that the externality will be concentrated in a style's twin, and not be dispersed across all other risky assets.

past performance. Finally, the style returns to fundamental value because of selling by fundamental traders, because of bad news about its own fundamentals, or most interestingly, because of good news in a competing style Y, which draws attention and investment dollars away from X.

In some cases, the cycles we describe may be reinforced by academic work analyzing the historical performance of a style. It is noteworthy that Banz' (1979) study on the outperformance of small cap stocks was followed by several years of strikingly good returns on that style. Our model explains this by saying that Banz' study attracted the attention of switchers, who diverted funds to small stocks, pushing them higher, thus drawing in yet more switchers and leading to a long period of superior performance. Of course, our model also predicts that these good returns should eventually be reversed, and it is interesting that after 1983, small cap stocks experienced two decades of lackluster returns.

More radically, cycles may be set in motion by data snooping, as analysts looking through historical data identify abnormal returns. When analysts succeed in convincing investors that they have found strategies earning true superior returns, they will recruit new resources to the strategies, thereby confirming the anomaly, at least over a period of time. Perhaps the discovery of the size effect in the 1970's is an example of such creation of a style out of what might have been a fluke in the data.

The externality from style switching may be helpful in interpreting other recent evidence. During 1998 and 1999, value stocks performed extremely poorly by historical standards, lagging both growth stocks and the broad index by a significant margin. As Chan, Karceski, and Lakonishok (2000) show, this poor performance occurred despite the fact that the earnings growth of value stocks over this period was as high as that of growth stocks, and if anything unusually good by historical standards. In other words, the poor performance of value portfolios cannot be easily linked to their fundamentals. A more natural explanation comes from our theory: the poor performance of value stocks in 1998-1999 might have been due to the spectacular performance of large growth stocks which generated large flows of funds – unrelated to fundamentals – into these stocks and out of value, the obvious competing style.

Another example comes, once again, from the historical performance of small stocks. Siegel (1999) argues that one reason for the vastly superior performance of small stocks relative to large stocks during 1975-1983 was the dismal performance of the "Nifty Fifty" large cap stocks in 1973-74. The demise of these high profile large stocks left investors disenchanted with the large stock style and generated a flow of funds into the competing style, small stocks, triggering a small stock cycle. A competing increase in the relative demand for large stocks, prompted by the rise of indexation and institutional investing more generally, may have arrested this wave of high small stock returns. According to Gompers and Metrick (2001), institutional investors prefer large stocks and their ownership of these stocks has increased rapidly in the last 20 years. This increase in demand for the competing style may be one reason for the poor relative performance of small stocks after 1983.

4 The Behavior of Asset Prices

We now present a systematic analysis of the effect of switcher flows on asset prices. We summarize the predictions of our model in a series of propositions, focusing on predictions that are largely unique to our framework.

To illustrate the propositions, we use simulated data. As in Section 3, we take $n = 50$, so that there are 100 stocks, the first 50 of which are in style X and the last 50 in style Y. For the parameter values in (21) and (22), we use the price formulae in (17) and (19) to simulate long time series of prices for the 100 assets.

4.1 Comovement Within Styles

Since switcher demand for securities is expressed at the level of a style, the prices of assets that are in the same style comove more than the assets' cash-flow fundamentals do. If style X has had superior past performance, switchers invest more in *all* securities in style X, pushing their prices up together. This coordinated demand generates comovement over and above that induced by cash-flow news. More formally, we can prove[12]

Proposition 1: If two assets i and j, $i \neq j$, belong to the same style, then

$$\text{corr}(\Delta P_{i,t}, \Delta P_{j,t}) > \text{corr}(\Delta D_{i,t}, \Delta D_{j,t}).$$

In our simulated data, the correlation matrix of returns is

$$\text{corr}(\Delta P_{1,t}, \ldots, \Delta P_{2n,t}) = \begin{pmatrix} \widehat{A} & \widehat{B} \\ \widehat{B} & \widehat{A} \end{pmatrix}, \tag{23}$$

with

$$\widehat{A} = \begin{pmatrix} 1 & 0.46 & \cdots & 0.46 \\ 0.46 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0.46 \\ 0.46 & \cdots & 0.46 & 1 \end{pmatrix}, \quad \widehat{B} = \begin{pmatrix} -0.16 & \cdots & \cdots & -0.16 \\ \vdots & & \ddots & \vdots \\ \vdots & & \ddots & \vdots \\ -0.16 & \cdots & \cdots & -0.16 \end{pmatrix}.$$

The average correlation between a pair of assets i and j which are in the *same* style is 0.46 in the presence of switchers; this is indeed higher than the cash-flow correlation of 0.31, reported in (21).

[12]Proofs of all propositions are in the Appendix.

Our model suggests that securities with unrelated cash-flow fundamentals may move together simply because they belong to the same style. Put differently, there is a common factor in returns to such securities even if there is no common factor in their fundamentals. More generally, even if the fundamentals of the securities in a particular style are correlated – as in our numerical example – the common factor in their returns is generally stronger than the common factor in their fundamentals. Moreover, changes in the value of the return factor need not be highly correlated with changes in the value of the cash-flow factor. Since this period's returns are in part driven by switcher flows and hence by past returns, they need not line up well with this period's cash-flow news.

This last point is illustrated by the two regressions

$$R_{X,t} = -0.005 + 1.00 R_{M,t} + 0.48 R_{S,t} + u_{X,t},\ R^2 = 92\%, \tag{24}$$

$$R_{X,t} = -0.005 + 1.00 \Delta D_{M,t} + 0.48 \Delta D_{S,t} + u_{X,t},\ R^2 = 50\%, \tag{25}$$

where

$$\begin{aligned} R_{X,t} &= \frac{1}{n/2} \sum_{l=1}^{n/2} \Delta P_{l,t}, \\ R_{M,t} &= \frac{1}{2n} \sum_{l=1}^{2n} \Delta P_{l,t}, \\ R_{S,t} &= \frac{1}{n/2} \sum_{l=n/2+1}^{n} \Delta P_{l,t} - \frac{1}{n/2} \sum_{l=3n/2+1}^{2n} \Delta P_{l,t}. \\ \Delta D_{M,t} &= \frac{1}{2n} \sum_{l=1}^{2n} \varepsilon_{l,t}, \\ \Delta D_{S,t} &= \frac{1}{n/2} \sum_{l=n/2+1}^{n} \varepsilon_{l,t} - \frac{1}{n/2} \sum_{l=3n/2+1}^{2n} \varepsilon_{l,t}. \end{aligned}$$

Here, $R_{X,t}$ is the return on a portfolio consisting of half of the n stocks in category X; $R_{M,t}$ is the market factor, the average return of all $2n$ stocks; and $R_{S,t}$ is a style factor, constructed as the return on a portfolio of the remaining stocks in category X minus the return on half the stocks in category Y. The reason we use only half the stocks in a style is to ensure that $R_{X,t}$ and $R_{S,t}$ are constructed using different stocks and hence that spurious correlation is avoided. $\Delta D_{M,t}$ and $\Delta D_{S,t}$ are a market cash-flow factor and a style cash-flow factor respectively, constructed in the same way as the return factors.

Regression (24) demonstrates that there is a strong common factor in the returns of X; however, the sharp drop in R^2 in regression (25) shows that as claimed, the return of style X does not line up well with cash-flow news about X.

The idea that style investing can generate comovement in returns unrelated to comovement in fundamentals has significant implications for the interpretation of security returns.

Fama and French (1993) show that there is a striking common factor in the returns of value stocks as well as a clear common component in small stock returns. The simplest rational pricing view of this comovement holds that it must be due to common factors in the underlying earnings of small stocks and value stocks. Fama and French (1995) test this explanation and obtain surprising results. Although they do find some evidence of a common factor in the fundamentals of small stocks, as well as in value stock fundamentals, these fundamental factors are weaker than the factors in returns. More importantly, there is little evidence that the return factors are driven by the fundamental factors.[13]

While these results do not sit well with the view that return comovement is always driven by cash-flow comovement, they emerge very naturally from a style investing perspective. This view argues that, to the extent that small firms belong to a style – because size is a characteristic defining a style – they will move together by virtue of fund flows in and out of that style. Even if there is a common component in the earnings of small firms for some reason, the common component in their returns should be much more pronounced. This is exactly Fama and French's (1995) finding.[14]

In the case of value stocks, excess comovement can arise in two distinct ways in our framework. The more obvious mechanism relies on "value stocks" being a style in itself; a direct application of Proposition 1 then predicts a stronger common factor in the returns of value stocks than in their cash flows. An alternative view is that industries are the most important styles and therefore exhibit excess comovement. Since value stocks belong to industries which have fallen out of favor among switchers, they comove excessively simply by inheriting that property from industry styles. Either mechanism explains Fama and French's results for value stocks.

Other evidence is also consistent with this analysis. Pindyck and Rotemberg (1990) find comovement in the prices of different commodities over and above what can be explained by economic fundamentals. Lee, Shleifer and Thaler (1991) find that the prices of closed-end mutual funds move together even when their fundamentals are unrelated. In the language of the present model, if investors classify all commodities into a "commodity" style and all closed-end funds into a "closed-end fund" style, and then move money in and out of these styles, the coordinated demand induces a common component in returns even when the

[13]In principle, changes in discount rates can also generate comovement. However, changes in interest rates or risk aversion induce a common factor in the returns of *all* stocks, and do not explain why a particular group of stocks comoves. A common factor in news about the risk of the assets in a style may be a source of comovement for those assets, but there is no direct evidence to support such a mechanism in the case of small or value stocks.

[14]Another prediction of our model is that keeping the cash-flow covariance matrix constant, an increase in the importance of style investing should increase the fraction of a stock's volatility that is due to common, rather than idiosyncratic shocks. Campbell et al. (2001) find that over the past three decades, firm-specific volatility has risen relative to total volatility. They argue that this is most likely due to the fact that the cash-flow covariance matrix *has* changed, with firm-specific cash-flow news becoming more volatile.

fundamentals of the different assets have little in common.

Also relevant are the findings of Froot and Dabora (1999), who study "Siamese twin" stocks such as Royal Dutch and Shell. These stocks are claims to the same cash-flow stream, but are primarily traded in different locations: Royal Dutch in the U.S. and Shell in the U.K. In a frictionless market, these stocks should move together. Froot and Dabora show, however, that Royal Dutch is more sensitive to movements in the U.S. market while Shell comoves more with the U.K. market. A style-based perspective provides a natural explanation: Royal Dutch, a member of the S&P 500, is buffeted by the flows of investors for whom the S&P 500 is a style and therefore comoves more with this index. For the same reason, Shell, a member of the FTSE index, comoves more with that index.[15]

Proposition 1 is driven by our assumption that investors classify assets into styles and allocate funds at the style level. Traditional models of positive feedback trading in which the feedback occurs at the individual asset level do *not* deliver Proposition 1, nor do conventional learning models in which the learning occurs asset by asset. In these models, asset returns are only as correlated as the underlying cash flows.

One class of models that *can* deliver excess comovement are models of portfolio insurance, as analyzed by Grossman and Zhou (1996) among others. These papers argue that there are many investors – often called "insurers" – whose wealth must not fall below some fixed fraction of their initial capital. When the stock market goes up (down), they effectively become less (more) risk averse and change their exposure to risky assets. Since changes in insurer risk aversion lead to a coordinated demand shock across all assets, such models can indeed explain why the market factor in returns might be stronger than the market factor in cash flows. They are hard-pressed, however, to explain why there should be a common factor in the returns of a *subset* of stocks – small stocks, say – that is unrelated to any common factor in their cash flows.

Kyle and Xiong (2001) propose a theory of comovement based on the idea that financial intermediaries experience wealth effects. When intermediaries suffer trading losses, their risk-bearing capacity is reduced, leading them to sell assets across the board and inducing correlation in fundamentally unrelated securities. This model seems appropriate for understanding why apparently unrelated assets trading in different countries comove strongly in times of financial crisis, such as August 1998. It is less plausible an explanation of a common factor in small stock returns.

Finally, in their study of closed-end funds, Lee et al. (1991) propose another view of comovement. Their view is related to our own, in that they assume that uninformed demand

[15] Another possible application of our model is to the common factors in liquidity documented by Chordia, Roll, and Subrahmanyam (2000). If flows of funds in and out of a stock affect the liquidity of a stock, style investing becomes one possible source of these common factors.

shifts affect prices, but it is nevertheless distinct. They argue that some groups of securities may only be held by a particular subset of all investors, such as individual investors. As these investors' risk aversion or sentiment changes, they change their exposure to the risky assets that they hold, inducing a common factor in the returns to these securities. In other words, this theory predicts that there will be a common factor in the returns of securities that are held primarily by a specific class of investors. This is distinct from our own theory, which predicts a common factor in the returns of securities that many investors classify as a style, even if these securities are in *all* investors' portfolios.

Lee et al.'s (1991) theory seems well-suited to explaining why small stocks and closed-end funds comove: both of these asset classes are held almost entirely by individual investors. Indeed, style investing is a less plausible explanation of this fact, since small stocks and closed-end funds do not form a natural single style. On the other hand, style investing may be a better way of thinking about the common factor in value stocks, since there is no evidence that these securities are held primarily by a particular investor class.

The style investing view of comovement has other predictions and implications for the interpretation of empirical facts. Not only should stocks within a style comove more than their fundamentals do, but stocks that *enter* a style should comove more with the style after they are added to it than before. For example, a stock that is added to an index such as the S&P 500 should comove more with the index after it is added than before. Changes in the pattern of comovement after a security is added to a style provide one of the more unique empirical predictions of our theory. More formally, we can prove:

Proposition 2: Suppose that asset j, not previously a member of style X, is reclassified as belonging to X. Then $cov(\Delta P_{j,t}, \Delta P_{X,t})$ increases after j is added to style X.

In our analysis so far, we have taken assets 1 through n to be in style X, and assets $n+1$ through $2n$ to be in style Y. In our simulated data for this economy, we find that for any asset j not in style X, in other words for $j = n+1, \ldots, 2n$,

$$\text{cov}(\Delta P_{j,t}, \Delta P_{X,t}) = -0.21.$$

Suppose now that asset 1 is reclassified into style Y and that asset $n+1$ is reclassified into style X. We now recompute asset $n+1$'s covariance with style X. In other words, we keep the cash-flow covariance matrix fixed, and use equation (17) to simulate prices as before, the only difference being the new composition of styles X and Y. We find that

$$\text{cov}(\Delta P_{n+1,t}, \Delta P_{X,t}) = 0.10.$$

In other words, stock $n+1$'s covariance with style X does indeed increase after it is added to that style.[16]

[16]Note that there will be a spurious increase in $\text{cov}(\Delta P_{n+1,t}, \Delta P_{X,t})$ arising from the fact that after

Interesting evidence about stocks entering new styles comes from European equity markets. Rouwenhorst (1999) notes that while money managers have traditionally allocated funds to European stocks at the country level, a growing number of them have started allocating funds by industry instead. Our model predicts that such a shift would make industry factors in returns relatively stronger and country factors relatively weaker. This is exactly the finding of Baca et al. (2000) and Cavaglia et al. (2000), who examine the importance of industry and country factors in European stock returns over time. Moreover, Rouwenhorst (1999) notes that when the shift to industry-level allocation began, country factors were still more important that industry factors. This suggests that the shift in allocation strategy generated the change in the structure of returns, rather than the other way around.[17]

In the data, portfolio premia are often associated with comovement: small stocks and value stocks have each had high average returns and they both exhibit comovement. One interpretation is that the comovement represents some kind of systematic risk which is then compensated by the observed premium. Our analysis suggests a different view. If an investment strategy is found to earn a premium, it may become labelled a "style," and mutual fund managers and institutional money managers will create products to facilitate investing in that style. The resulting flow of funds into the style generates comovement. In other words the premia may lead to comovement, rather than the comovement leading to premia.

Even if the comovement in small stocks and value stocks does represent a form of systematic risk which ultimately generates a premium, our analysis suggests that at least part of this risk is induced by investors themselves through their style-based strategies, rather than being created by comovement in firm fundamentals. The risk is systematic even if it is not fundamental (De Long et al. 1990b).

4.2 Comovement Across Styles

Two assets in the *same* style, then, will be more correlated than their underlying fundamentals. Interestingly, the opposite is true of two assets in *different* styles, asset i in style X, say, and asset j in style Y. Such assets will be *less* correlated than their underlying fundamentals. The reason for this is the externality generated by switchers: a good return for style X leads to a flow out of Y and into X, driving the styles in opposite directions,

reclassification, $\Delta P_{n+1,t}$ enters into the computation of $\Delta P_{X,t}$. The change from -0.21 to 0.10, however, is far in excess of this mechanical effect.

[17]Proposition 2 may also help us differentiate the two views of value stock comovement suggested earlier. If it arises because "value stocks" is itself a style, the proposition predicts that stocks that become value stocks will comove more after entering that category than before. If it arises because industries are styles, there will be no increased comovement after an industry enters the value category. Daniel and Titman's (1997) evidence is more supportive of the latter view: stocks in the value category today comove roughly as much as they did five years earlier.

and lowering the correlation between them. More formally, we can prove:

Proposition 3: If assets i and j are in different styles, then

$$\text{corr}(\Delta P_{i,t} - \Delta P_{M,t}, \Delta P_{j,t} - \Delta P_{M,t}) < \text{corr}(\Delta D_{i,t} - \Delta D_{M,t}, \Delta D_{j,t} - \Delta D_{M,t}),$$

where

$$\begin{aligned} \Delta P_{M,t} &= \frac{1}{2n} \sum_{l=1}^{2n} \Delta P_{l,t} \\ \Delta D_{M,t} &= \frac{1}{2n} \sum_{l=1}^{2n} \Delta D_{l,t}. \end{aligned}$$

The prediction is made in terms of market-adjusted returns and not raw returns. It is tempting to think that the externality would make returns on small stocks and large stocks and returns on value stocks and growth stocks pairwise less correlated than their fundamentals. However, reality may be more complicated because of overlap between styles. Competition between value and growth suggests that their returns are less correlated than their fundamentals, but both value stocks and growth stocks are part of the overall U.S. stock market, itself a style. By Proposition 1, this would tend to make value and growth stocks *more* correlated than their cash flows. In view of this complication, we make our prediction in terms of market-adjusted returns. In other words, we predict that the market-adjusted returns on value and growth stocks are less correlated than the fundamentals of value and growth stocks, in turn adjusted for market fundamentals.

In our simulated data, we find that for any asset i in X and any asset j in Y,

$$\begin{aligned} \text{corr}(\Delta P_{i,t} - \Delta P_{M,t}, \Delta P_{j,t} - \Delta P_{M,t}) &= -.30 \\ \text{corr}(\Delta D_{i,t} - \Delta D_{M,t}, \Delta D_{j,t} - \Delta D_{M,t}) &= -.13, \end{aligned}$$

showing that the market-adjusted returns are indeed less correlated than the market-adjusted cash flows. Figure 3, which plots the prices of styles X and Y over a 100 period segment of the simulated data, provides another view of the same phenomenon: the price paths of styles X and Y tend to move in opposite directions.

One potential application of our result that different styles exhibit insufficient comovement is to stocks and bonds themselves. A number of authors (Summers 1983, Barsky 1989) have noted that these two broad asset classes appear to comove too little when one considers that any news about future riskfree rates should tend to push them in the same direction: the 1970's for example, were characterized by low stock market valuations in spite of low real interest rates, while the 1980's brought a sustained rise in the stock market in spite of much higher real rates. Another striking example of divergence between bonds and stocks occurred in the fall of 1998 following Russia's devaluation of the rouble and default on outstanding debt: bond prices rose sharply while the stock market fell.

Our model sheds light on such puzzling delinkages between the stock and bond markets through flows of funds from one asset class to another which push them in opposite directions. When applied to the fall of 1998, our model effectively captures a "flight to quality" phenomenon often discussed in the financial press.

4.3 Own- and Cross-autocorrelations

The presence of switchers in the market makes returns on a style positively autocorrelated in the short run, and negatively autocorrelated in the long run: a good return for style X draws in switchers who push its price up again next period, inducing positive autocorrelation. The price swing is eventually reversed in the long run, creating mean-reversion.

Our model's predictions about own-autocorrelations are not unique to our framework: they could also arise from traditional positive feedback trading models or conventional learning models so long as the assets in a style share a common cash-flow factor. However, the relative performance aspect of our framework, and in particular the externality that it creates, leads to more unique predictions about *cross*-autocorrelations across styles, namely that they should be negative in the short run and positive in the long run. A good return on style X at time t generates outflows from Y into X, pushing Y's price down at time $t+1$. In the long run, Y's price recovers, generating positive cross-autocorrelations at longer lags. In summary, we can show:

Proposition 4: For some $K \geq 1$,

$$\begin{aligned} \text{(i) } \operatorname{corr}(\Delta P_{X,t}, \Delta P_{X,t-k}) &> 0, \; 1 \leq k \leq K \\ \operatorname{corr}(\Delta P_{X,t}, \Delta P_{X,t-k}) &< 0, \; k = K+1, \\ \text{and (ii) } \operatorname{corr}(\Delta P_{X,t}, \Delta P_{Y,t-k}) &< 0, \; 1 \leq k \leq K \\ \operatorname{corr}(\Delta P_{X,t}, \Delta P_{Y,t-k}) &> 0, \; k = K+1. \end{aligned}$$

Table 1 shows the magnitude of these own- and cross-autocorrelations for our particular numerical example.[18] The first order own-autocorrelation is 0.5, while the correlation of returns nine lags apart is -0.18.

The available evidence on own-autocorrelations is generally supportive of part (i) of Proposition 4. Lo and MacKinlay (1988) and Poterba and Summers (1988) find U.S. monthly

[18] It is easily proved that $\operatorname{corr}(\Delta P_{X,t}, \Delta P_{X,t-k}) = -\operatorname{corr}(\Delta P_{X,t}, \Delta P_{Y,t-k})$ for $k \geq 1$, a pattern that is clearly visible in the table.

stock returns to be positively autocorrelated at the first lag and negatively autocorrelated thereafter. Cutler, Poterba, and Summers (1991) show that the monthly returns of international stock and bond indices, as well as of real estate and commodity markets are positively autocorrelated at lags of up to a year, and negatively autocorrelated thereafter. Finally, Lewellen (2000) finds that monthly returns on industry and size-sorted portfolios are positively autocorrelated at the one month lag and negatively autocorrelated after that.

Evidence on part (ii) of Proposition 4 is harder to come by. Lewellen (2000) finds that industry portfolios are negatively cross-autocorrelated at the first 12 monthly lags, and that size-sorted portfolios are negatively cross-autocorrelated at monthly lags two through 12, but positively cross-autocorrelated at the first monthly lag. While this evidence may initially appear somewhat supportive of our prediction, it in fact represents something of a puzzle. Our prediction is that over those horizons where styles have negative *own*-autocorrelations – as they do from the second monthly lag in the case of industry and size portfolios – *cross*-autocorrelations should be *positive*, not negative.

It is possible that our model's fit with the data may improve after taking style overlaps into account: stocks will be affected both by flows into the stock market as a whole, as well as by intra stock market flows between styles. A more robust version of Proposition 5 would therefore be in terms of market adjusted returns:

Proposition 5: For some $K \geq 1$,

$$\begin{aligned}
\text{(i) } \operatorname{corr}(\Delta P_{X,t} - \Delta P_{M,t}, \Delta P_{X,t-k} - \Delta P_{M,t-k}) &> 0,\ 1 \leq k \leq K \\
\operatorname{corr}(\Delta P_{X,t} - \Delta P_{M,t}, \Delta P_{X,t-k} - \Delta P_{M,t-k}) &< 0,\ k = K+1, \\
\text{and (ii) } \operatorname{corr}(\Delta P_{X,t} - \Delta P_{M,t}, \Delta P_{Y,t-k} - \Delta P_{M,t-k}) &> 0,\ 1 \leq k \leq K \\
\operatorname{corr}(\Delta P_{X,t} - \Delta P_{M,t}, \Delta P_{Y,t-k} - \Delta P_{M,t-k}) &< 0,\ k = K+1.
\end{aligned}$$

Table 1 presents the magnitudes of these own- and cross-autocorrelations in simulated data.

4.4 Asset-level Momentum and Contrarian Strategies

In this section, we analyze the profitability of momentum and value strategies that are implemented at the level of individual securities. An individual stock-level momentum strategy ranks all stocks on their return in the previous period, buys those stocks that did better than average and sells those that did worse. It can be implemented through

$$N_{i,t} = \frac{1}{2n} \left[\Delta P_{i,t} - \Delta P_{M,t} \right],\ i = 1, \ldots, 2n. \tag{26}$$

An individual stock-level value strategy buys (sells) those stocks which are trading below (above) fundamental value:

$$N_{i,t} = \frac{1}{2n}\left[P^{*}_{i,t} - P_{i,t}\right], \; i = 1, \ldots, 2n. \tag{27}$$

In the presence of switchers, we expect both individual stock-level momentum and individual stock-level value strategies to be profitable. If a stock performed well last period, there is good chance that the outperformance was due to the stock's being a member of a "hot" style which is enjoying inflows from switchers. If so, the style is likely to keep attracting inflows from switchers next period, making it likely that the stock itself also does well next period.

Similarly, a stock trading below its fundamental value may be in this predicament because it is a member of a style that is currently unpopular with switchers and is suffering fund outflows. If so, we expect the style and all the stocks in it to eventually correct back up to fundamental value, bringing high returns to a value strategy.[19] Specifically, we can prove

Proposition 6: The individual stock-level momentum strategy in (26) and the individual stock-level value strategy in (27) have strictly positive expected returns in the presence of switchers.

Table 2 confirms that, in our simulated data, both these strategies offer attractive Sharpe ratios.[20]

Taken together with our earlier results on comovement, Proposition 6 suggests that our simulated data may be able to match the regression evidence on value stock returns and earnings very precisely. Fama and French (1992, 1993, 1995) document a four-part puzzle: first, value stocks earn a premium not captured by the CAPM; second, value stocks comove, and loadings on a specific factor, christened the HML factor, can capture the value premium; third, the common factor in value stock returns is stronger than the common factor in their earnings; and finally, shocks to the common factor in value stock returns are only weakly correlated with shocks to the common factor in value stock earnings news.

To see if our simulated data can replicate this evidence, we run the following three regressions; as usual, we think of X and Y as two fixed styles, such as old economy and new

[19] In our model, the small stock premium is much smaller than the value stock premium; indeed the only reason there is any premium to small stocks at all is because they often have low prices relative to fundamentals and therefore inherit part of the value premium. Any small stock premium that appears in a small sample of the simulated data is therefore largely reversed in the long run. This is not so for the value premium, which exists even in the long run in our model, for the reasons outlined in this section.

[20] The Sharpe ratio we compute is the mean one period change in wealth from implementing the strategy divided by the standard deviation of the one period change in wealth.

economy stocks:

$$
\begin{aligned}
R_{Val,t} &= 0.044 + 1.06 R_{M,t} + u_{Val,t}, && (28)\\
R_{Val,t} &= -0.009 + 1.00 R_{M,t} + 0.48 R_{S,t} + u_{Val,t},\ R^2 = 92\%, && (29)\\
R_{Val,t} &= 0.033 + 1.00 \Delta D_{M,t} + 0.48 \Delta D_{S,t} + u_{Val,t},\ R^2 = 50\%. && (30)
\end{aligned}
$$

The regression variables are constructed in the following way. Each period, we rank all risky assets on the difference between their prices and fundamental values, $P_{i,t} - P^*_{i,t}$. The 50 percent of stocks with lower such values, we call value stocks, and split them randomly into two equally-sized groups, $VAL_{A,t}$ and $VAL_{B,t}$. The remaining stocks we call growth stocks, and also split them randomly into $GWT_{A,t}$ and $GWT_{B,t}$. We then define

$$
\begin{aligned}
R_{Val,t} &= \frac{1}{n/2} \sum_{l_t \epsilon VAL_{A,t}} \Delta P_{l_t,t},\\
R_{M,t} &= \frac{1}{2n} \sum_{l=1}^{2n} \Delta P_{l,t},\\
R_{S,t} &= \frac{1}{n/2} \sum_{l_t \epsilon VAL_{B,t}} \Delta P_{l_t,t} - \frac{1}{n/2} \sum_{l_t \epsilon GWT_{B,t}} \Delta P_{l,t},\\
\Delta D_{M,t} &= \frac{1}{2n} \sum_{l=1}^{2n} \varepsilon_{l,t},\\
\Delta D_{S,t} &= \frac{1}{n/2} \sum_{l_t \epsilon VAL_{B,t}} \varepsilon_{l_t,t} - \frac{1}{n/2} \sum_{l_t \epsilon GWT_{B,t}} \varepsilon_{l_t,t}.
\end{aligned}
$$

In words, $R_{Val,t}$ is the return on a portfolio consisting of half the available universe of n value stocks; $R_{M,t}$ is the market factor, the average return of all $2n$ stocks; and $R_{S,t}$ is a style factor, analogous to the HML factor, constructed as the return on a portfolio of the remaining value stocks minus the return on half the available growth stocks. We sometimes use only half the stocks in a style to ensure that $R_{Val,t}$ and $R_{S,t}$ are constructed using different stocks and hence that spurious correlation is avoided. Finally, $\Delta D_{M,t}$ and $\Delta D_{S,t}$ are a market fundamental factor and a style fundamental factor respectively, constructed in the same way as the return factors.

The intercept in (28) confirms that the value portfolio $R_{Val,t}$ earns an anomalously high return, as judged by the CAPM. The positive slope on $R_{S,t}$ in (29) shows that there is a common factor in the returns of value stocks, while the tiny intercept shows that loadings on this factor can help capture the value premium. That the common factor in returns is stronger than the common factor in earnings follows directly from Proposition 1. Finally, the drop in R^2 between (29) and (30) shows that the common factor in fundamentals lines up poorly with the common factor in returns.

It is important to be clear as to which predictions are unique to our framework, and which are shared with other models. The first part of the value puzzle – the existence of a value premium – can also be explained by the feedback models of De Long et al. (1990a) and Hong and Stein (1999), by the more psychological models of Barberis, Shleifer, Vishny (1998), Daniel, Hirshleifer, Subrahmanyam (2001), and Barberis and Huang (2001), as well as by the learning model of Lewellen and Shanken (2002). Moreover, the mechanism used in this paper – an excessively negative reaction on the part of some investors to poor prior performance – is similar to that used by De Long et al. (1990a) and Hong and Stein (1999).

These alternative models can potentially also explain the second part of the puzzle so long as they introduce some mechanism for making value stocks comove: if value stocks comove, the intercept on $R_{S,t}$ in (29) will be positive and the $R_{S,t}$ term will therefore help soak up the unexplained intercept in (28). A simple way of generating such comovement, suggested by Daniel and Titman (1997), is to have industry cash-flow factors. As industries fall out of favor among some investors and become value industries, value stocks comove simply by inheriting the comovement from industries. This mechanism can certainly operate in our model too, but we also offer the alternative possibility that comovement in value stocks arises because "value stocks" it a style in itself, and not necessarily because of industry cash-flow factors.

The models of Barberis, Shleifer, and Vishny (1998), Barberis and Huang (2001), and Lewellen and Shanken (2002) have trouble, however, on the fourth part of the puzzle, since they tie returns closely to cash flows, making it hard to generate a low R^2 in (30). More important, though, all the alternative models fail on the third part of the puzzle, namely the presence of a stronger common factor in returns than in cash flows: since the central mechanism in all these papers, whether learning from cash flows or overconfidence about private information, operates on an asset by asset basis, returns are only as correlated as the underlying cash flows.

4.5 Style-level Momentum and Contrarian Strategies

In order to make other predictions that distinguish our framework, we introduce some additional investment strategies. First, we consider style-level versions of (26) and (27). A style-level momentum strategy buys into *styles* with good recent performance and avoids styles that have done poorly:

$$\begin{aligned} N_{i,t} &= \frac{1}{2n}\left[\frac{\Delta P_{X,t} - \Delta P_{Y,t}}{2}\right],\ i \epsilon X, \\ N_{j,t} &= \frac{1}{2n}\left[\frac{\Delta P_{Y,t} - \Delta P_{X,t}}{2}\right],\ j \epsilon Y. \end{aligned} \tag{31}$$

A style-level value strategy buys into styles trading below fundamental value and shorts the remaining styles:

$$N_{i,t} = \frac{1}{2n}\left[P^*_{X,t} - P_{X,t}\right],\ i\epsilon X, \tag{32}$$
$$N_{j,t} = \frac{1}{2n}\left[P^*_{Y,t} - P_{Y,t}\right],\ j\epsilon Y.$$

Finally, we also consider "within-style" versions of (26) and (27). In a within-style momentum strategy, the investor buys those stocks which outperformed their style last period and sells those which underperformed their style. It can be implemented through

$$N_{i,t} = \frac{1}{2n}\left[\Delta P_{i,t} - \Delta P_{X,t}\right],\ i\epsilon X, \tag{33}$$
$$N_{j,t} = \frac{1}{2n}\left[\Delta P_{j,t} - \Delta P_{Y,t}\right],\ j\epsilon Y.$$

Correspondingly, a within-style value strategy buys (sells) stocks which are trading at a larger discount (premium) of price to fundamental value than their style:

$$N_{i,t} = \frac{1}{2n}(\left[P^*_{i,t} - P_{i,t}\right] - \left[P^*_{X,t} - P_{X,t}\right]),\ i\epsilon X, \tag{34}$$
$$N_{j,t} = \frac{1}{2n}(\left[P^*_{j,t} - P_{j,t}\right] - \left[P^*_{Y,t} - P_{Y,t}\right]),\ j\epsilon Y.$$

These new strategies allow us to make predictions that are unique to our framework. In particular, we can prove:

Proposition 7: (i) Style-level momentum and value strategies offer Sharpe ratios that are greater than or equal to those of their individual stock-level counterparts; and (ii) within-style momentum and value strategies are unprofitable: their expected return is zero.

Table 2 reports the Sharpe ratios of both style-level and within-style momentum and value strategies, illustrating the proposition for one particular set of parameter values.

The intuition behind Proposition 7 is straightforward: since mispricing in our model occurs at the level of a style, a strategy designed to exploit this mispricing must be at least as effective when implemented at the style level as it is when implemented at the individual asset level. Moreover, precisely because mispricing is a style-level phenomenon, style-neutral strategies like the within-style strategies will not be able to exploit any mispricing and will remain unprofitable.

Proposition 7 does not hold in traditional positive feedback trading models where the feedback occurs at the individual asset level. In this case, the most effective momentum strategy will buy individual stocks which outperformed last period, in anticipation of further purchases by positive feedback traders. A strategy that simply buys outperforming styles is a less efficient way of picking out future winners and hence offers much lower Sharpe ratios. On the other hand, since mispricing occurs at the individual asset level in these models, even style-neutral momentum strategies can be profitable, in contrast to Proposition 7.

In taking Proposition 7 to the data, it is worth keeping in mind that it relies on our simplifying but strong assumption that all noise trading occurs at the style level. In practice, at least some noise trading is likely to be an individual stock-level phenomenon. In this case, we still expect style-level strategies to do well, although not necessarily better than individual stock-level strategies; and we expect within-style strategies to do less well, although their average return need not be as low as zero.

What evidence there is on style momentum strategies provides some support for style investing. In line with part (i) of Proposition 7, a number of papers find that some style-level momentum strategies have earned average returns as impressive as those of individual stock-level momentum. Moskowitz and Grinblatt (1999) show that a momentum strategy based on industry portfolios is profitable.[21] Lewellen (2000) investigates a momentum strategy using size-sorted portfolios, and finds it to be successful. Chen (2001) forms portfolios based on independent sorts on size, book-to-market, and the dividend yield, and finds that a momentum strategy implemented on these portfolios earns high returns; he also shows that these profits are not simply a manifestation of individual stock momentum. Richards (1996) and Asness, Porter, and Stevens (1997) show that a momentum strategy works well when applied to country portfolios. Also consistent with our model, Haugen and Baker (1996) track returns on a large number of investment styles and show that a strategy which tilts towards styles with good prior performance earns very high risk-adjusted returns – higher than the returns on any one style. They successfully replicate their findings in out-of-sample tests in a number of international markets.

Evidence on the performance of style-based value strategies is more limited, but Asness, Porter, and Stevens (1997) show that a value strategy applied to country portfolios works well. The amount of work bearing directly on part (ii) of Proposition 7 is also small. However, some support comes from Moskowitz and Grinblatt (1999), who find that within-industry momentum strategies are unprofitable.

While Table 2 shows that both style momentum and value strategies are profitable, it leaves open the question of whether there are times when one of them is relatively more profitable than the other. In other words, would an arbitrageur want to put more weight on

[21] Grundy and Martin (2001) emphasize that the results of Moskowitz and Grinblatt (1999) depend heavily on the positive autocorrelation of monthly industry returns at the first lag. If a gap of a month is inserted between the portfolio formation period and the portfolio test period, individual stock-level momentum is profitable, while industry-level momentum is less so. This suggests that momentum in stock returns cannot be purely a style effect.

one strategy versus the other at different points of the style cycle?

One way to address this is to compute the optimal strategy for an arbitrageur who is clever enough to figure out the correct process followed by asset prices in our economy.

Proposition 8: The optimal strategy for an arbitrageur who knows that prices follow the process in (17) is given below. In deriving it, we restrict the arbitrageur to style-level strategies in which demand for all assets within the same style is identical.[22]

$$\begin{aligned} N^A_{i,t} &= \frac{c}{n}\left[\frac{\Delta P_{X,t}-\Delta P_{Y,t}}{2}-(1-\theta)\sum_{k=1}^{t-1}\theta^{k-1}\left(\frac{\Delta P_{X,t-k}-\Delta P_{Y,t-k}}{2}\right)\right],\ i\epsilon X \qquad (35)\\ N^A_{j,t} &= \frac{c}{n}\left[\frac{\Delta P_{Y,t}-\Delta P_{X,t}}{2}-(1-\theta)\sum_{k=1}^{t-1}\theta^{k-1}\left(\frac{\Delta P_{Y,t-k}-\Delta P_{X,t-k}}{2}\right)\right],\ j\epsilon Y, \end{aligned}$$

where c is a positive constant.[23]

If we use $N^{mom}_{i,t}$ and $N^{val}_{i,t}$ to denote the share demands of the style-level momentum and value strategies in equations (31) and (32) respectively, then (35) together with (17) imply that

$$N^A_{i,t} = 2c[N^{mom}_{i,t} + \phi(1-\theta)N^{val}_{i,t}]. \qquad (36)$$

Equation (36) implies that an arbitrageur would *not* want to "time" the momentum and value strategies over the course of the style cycle; instead, his optimal strategy is a constant combination of momentum and value. It is important to note that the arbitrageur *does* switch between styles X and Y themselves over the course of the cycle: if X has had many periods of poor returns, he will buy more of X in anticipation of a reversal in its price. However, equation (36) shows that even though the relative weight on X and Y changes over time, the relative weight on the momentum and value strategies does not.

Another striking result in Table 2 is that the style momentum strategy has a much higher Sharpe ratio than the style value strategy. Because switcher flows into styles are very persistent in our model and mispricings take a long time to correct, it makes more sense for an arbitrageur to ride with the switchers than to bet against them. For lower values of θ, switcher flows are less persistent and prices revert to fundamental value more quickly. This suggests that for low θ, the value strategy should be relatively more attractive. The expression in (36) confirms that as θ falls, value becomes more attractive relative to momentum in the sense that the optimal strategy places relatively more weight on value.[24]

[22] This restriction makes it easier to relate the optimal strategy to the style-level momentum and value strategies.

[23] The "A" superscript in these expressions stands for arbitrageur.

[24] This observation requires some computation, because ϕ is itself a function of θ. We find that ϕ, and therefore $\phi(1-\theta)$, is a decreasing function of θ.

5 Special Styles

An important style that deserves further discussion is indexation. In Section 4, we informally applied some of our propositions to the case where one of the styles is an index, but it may be helpful to restate our predictions about this style more explicitly. Setting $X = I$ and $Y = NI$ gives

Proposition 9: (i) Suppose that asset j, not previously a member of an index I, is reclassified as belonging to I. Then $cov(\Delta P_{j,t}, \Delta P_{I,t})$ increases after j is added to I; (ii) If asset i is in an index I while asset j is not, then

$$\text{corr}(\Delta P_{i,t} - \Delta P_{M,t}, \Delta P_{j,t} - \Delta P_{M,t}) < \text{corr}(\Delta D_{i,t} - \Delta D_{M,t}, \Delta D_{j,t} - \Delta D_{M,t});$$

(iii) For some $K \geq 1$,

$$\begin{aligned}
\text{corr}(\Delta P_{I,t}, \Delta P_{I,t-k}) &> 0, \; 1 \leq k \leq K \\
\text{corr}(\Delta P_{I,t}, \Delta P_{I,t-k}) &< 0, \; k = K+1, \\
\text{corr}(\Delta P_{I,t}, \Delta P_{NI,t-k}) &< 0, \; 1 \leq k \leq K \\
\text{corr}(\Delta P_{I,t}, \Delta P_{NI,t-k}) &> 0, \; k = K+1.
\end{aligned}$$

In words, a stock which is added to an index should comove more with the index after inclusion than before; the returns on a stock in an index should comove with the returns on a stock not in the index less than their fundamentals do; indices should have positive (negative) own-autocorrelations at short (long) horizons; and they should be negatively (positively) cross-autocorrelated with stocks outside the index at short (long) horizons. Moreover, given that the importance of indexing has grown over time, these phenomena should be stronger in more recent data samples. Barberis, Shleifer, and Wurgler (2001) examine some of these predictions empirically using data for the S&P 500 index, and find supportive evidence.

The idea that, when deciding whether to index, switchers compare indexers to active managers raises another issue. Active managers may strategically alter the proportion of stocks within the index that they hold so as to control the key variable that switchers pay attention to, namely performance relative to the index. For example, if an index enters a switcher-driven period of high returns, active managers may increase their holdings of stocks in the index, reducing their tracking error relative to the index in order to slow down outflows from their funds.

Another interesting issue arises with *price-dependent* styles, where the characteristic defining the style depends on price. Many common styles such as "small stocks" or "value stocks" fall into this category. When a style is price-dependent, its composition may change. Suppose that switchers have kicked off a long upswing in the price of small stocks relative to their fundamentals: they buy small stocks, pushing up their price, which attracts more switchers, and so on. After a while, some of the small stocks experience price increases so large that they cannot be considered small any more, and are no longer part of the small stock style.

This change in composition need not brake the evolution of the small stock style itself. However, it may mean that the degree of misvaluation experienced by any individual asset is *lower* than in the case where the style characteristic is not price-dependent: if a small stock becomes too highly valued relative to its fundamentals, it ceases to be a small stock and the buying pressure from switchers following the small stock style eases off, halting its ascent. This argument depends on the correlation between characteristic and price being negative: the higher a stock's price, the less likely it is to be a small stock. When this correlation is positive, misvaluation of individual stocks is more severe when the style is price-dependent than when it is not.

6 Conclusion

The model of financial markets discussed in this paper is in many ways similar to that proposed by Black (1986). On the one hand, financial markets in our economy are not efficient. Prices deviate substantially from fundamental values as styles become popular or unpopular. Such price deviations can easily look like bubbles. If an observer knows what is going on, there are substantial, though risky, profits to be made from a combination of contrarian and momentum trading. On the other hand, despite the fact that markets are inefficient, prices are extremely noisy. Patterns in security prices are complex, and may change significantly over time. Without knowing which style or model is favored, arbitrage is very risky and there are no consistent profits to be had. Moreover, the analysis is fraught with the danger of finding patterns where none exist. To some people such markets might even appear to be efficient.

Such markets are not entirely anarchic, however. They do exhibit long run pressures toward fundamentals. Moreover, there are empirical predictions that one can make about security prices in such markets, including excess comovement. In this paper, we have only begun to scratch the surface of such markets. It is likely that further predictions will emerge as we look at financial markets from the perspective of style investing.

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8 Appendix

Proof of Proposition 1: From equation (17), for any $i \epsilon X$,

$$\Delta P_{i,t+1} = \varepsilon_{i,t+1} + \frac{\Delta N^S_{X,t+1}}{\phi}, \tag{37}$$

where

$$\Delta N^S_{X,t+1} = \frac{\Delta P_{X,t} - \Delta P_{Y,t}}{2} - (1-\theta) \sum_{k=1}^{t-1} \theta^{k-1} (\frac{\Delta P_{X,t-k} - \Delta P_{Y,t-k}}{2})$$

is known at time t. This implies that for any $i, j \epsilon X$ with $i \neq j$,

$$\begin{aligned} \text{cov}(\Delta P_{i,t}, \Delta P_{j,t}) &= \psi_M^2 + \psi_S^2 + \frac{1}{\phi^2} \text{var}(\Delta N^S_{X,t}), \\ \text{var}(\Delta P_{l,t}) &= 1 + \frac{1}{\phi^2} \text{var}(\Delta N^S_{X,t}), \quad l \epsilon \{i, j\}. \end{aligned}$$

For such i and j we also have

$$\begin{aligned} \text{cov}(\Delta D_{i,t}, \Delta D_{j,t}) &= \psi_M^2 + \psi_S^2, \\ \text{var}(\Delta D_{l,t}) &= 1, \quad l \epsilon \{i, j\}, \end{aligned}$$

and this implies

$$\text{corr}(\Delta P_{i,t}, \Delta P_{j,t}) > \text{corr}(\Delta D_{i,t}, \Delta D_{j,t}).$$

Proof of Proposition 2: Suppose that asset $n+1$ is reclassified from style Y into style X, and that at the same time, asset 1 is reclassified from style X into style Y. Before reclassification, we have

$$\begin{aligned} \Delta P_{X,t+1} &= \varepsilon_{X,t+1} + \frac{\Delta N^S_{X,t+1}}{\phi} \\ \Delta P_{n+1,t+1} &= \varepsilon_{n+1,t+1} - \frac{\Delta N^S_{X,t+1}}{\phi}, \end{aligned}$$

where

$$\varepsilon_{X,t+1} = \frac{1}{n} \sum_{l \epsilon X} \varepsilon_{l,t+1}.$$

This implies that before reclassification,

$$\text{cov}(\Delta P_{n+1,t}, \Delta P_{X,t}) = \psi_M^2 - \frac{1}{\phi^2} \text{var}(\Delta N^S_{X,t}).$$

After reclassification, we have

$$\begin{aligned} \Delta P_{X,t+1} &= \varepsilon_{X,t+1} + \frac{\Delta N^S_{X,t+1}}{\phi} \\ \Delta P_{n+1,t+1} &= \varepsilon_{n+1,t+1} + \frac{\Delta N^S_{X,t+1}}{\phi}, \end{aligned}$$

which implies

$$\text{cov}(\Delta P_{n+1,t}, \Delta P_{X,t}) = \psi_M^2 + \frac{1-\psi_M^2}{n} + \frac{1}{\phi^2}\text{var}(\Delta N_{X,t+1}^S).$$

Therefore, $\text{cov}(\Delta P_{n+1,t}, \Delta P_{X,t})$ does indeed increase after addition.

Proof of Proposition 3: Since

$$\Delta P_{M,t+1} = \varepsilon_{M,t+1} = \frac{1}{2n}\sum_{l=1}^{2n}\varepsilon_{l,t+1},$$

we have, for $i\epsilon X$ and $j\epsilon Y$,

$$\begin{aligned} \Delta P_{i,t+1} - \Delta P_{M,t+1} &= \varepsilon_{i,t+1} - \varepsilon_{M,t+1} + \frac{\Delta N_{X,t+1}^S}{\phi} \\ \Delta P_{j,t+1} - \Delta P_{M,t+1} &= \varepsilon_{j,t+1} - \varepsilon_{M,t+1} - \frac{\Delta N_{X,t+1}^S}{\phi}. \end{aligned} \tag{38}$$

This implies

$$\begin{aligned} \text{cov}(\Delta P_{i,t+1} - \Delta P_{M,t+1}, \Delta P_{j,t+1} - \Delta P_{M,t+1}) &= \text{cov}(\varepsilon_{i,t+1} - \varepsilon_{M,t+1}, \varepsilon_{j,t+1} - \varepsilon_{M,t+1}) - \frac{1}{\phi^2}\text{var}(\Delta N_{X,t+1}^S) \\ \text{var}(\Delta P_{l,t+1} - \Delta P_{M,t+1}) &= \text{var}(\varepsilon_{i,t+1} - \varepsilon_{M,t+1}) + \frac{1}{\phi^2}\text{var}(\Delta N_{X,t+1}^S), \quad l\epsilon\{i,j\} \\ \text{cov}(\Delta D_{i,t+1} - \Delta D_{M,t+1}, \Delta D_{j,t+1} - \Delta D_{M,t+1}) &= \text{cov}(\varepsilon_{i,t+1} - \varepsilon_{M,t+1}, \varepsilon_{j,t+1} - \varepsilon_{M,t+1}) \\ \text{var}(\Delta D_{l,t+1} - \Delta D_{M,t+1}) &= \text{var}(\varepsilon_{i,t+1} - \varepsilon_{M,t+1}), \quad l\epsilon\{i,j\}. \end{aligned}$$

The proposition therefore follows if

$$-\text{cov}(\varepsilon_{i,t+1} - \varepsilon_{M,t+1}, \varepsilon_{j,t+1} - \varepsilon_{M,t+1}) < \text{var}(\varepsilon_{i,t+1} - \varepsilon_{M,t+1}). \tag{39}$$

Using

$$\varepsilon_{i,t+1} - \varepsilon_{M,t+1} = \frac{\psi_S f_{X,t+1} - \psi_S f_{Y,t+1}}{2} + \sqrt{1-\psi_M^2-\psi_S^2}(f_{i,t+1} - \frac{1}{2n}\sum_{l=1}^{2n} f_{l,t+1}),$$

it is easily checked that

$$\begin{aligned} \text{cov}(\varepsilon_{i,t+1} - \varepsilon_{M,t+1}, \varepsilon_{j,t+1} - \varepsilon_{M,t+1}) &= -(\frac{\psi_S^2}{2} + \frac{1-\psi_M^2-\psi_S^2}{2n}), \\ \text{var}(\varepsilon_{i,t+1} - \varepsilon_{M,t+1}) &= \frac{\psi_S^2}{2} + \frac{2n-1}{2n}(1-\psi_M^2-\psi_S^2), \end{aligned}$$

which means that (39) does indeed hold.

We now prove the following lemma, which will be useful in the remaining proofs.

Lemma: In any stationary equilibrium with $\theta > 0$, it must be true that

$$\begin{aligned} 0 &< \theta < 1 \\ \phi &> 1. \end{aligned}$$

Proof of Lemma: It is easily checked using (37) that $\rho_1 > \rho_2$, where ρ_1 and ρ_2 are defined in (13). Equation (18) then immediately implies that $\phi > 0$ in any stationary equilibrium.

From (17), we can write

$$\Delta P_{X,t+1} - \Delta P_{Y,t+1} = (\varepsilon_{X,t+1} - \varepsilon_{Y,t+1}) + \frac{\Delta P_{X,t} - \Delta P_{Y,t}}{\phi} - \frac{(1-\theta)}{\phi} \sum_{k=1}^{t-1} \theta^{k-1} (\Delta P_{X,t-k} - \Delta P_{Y,t-k}), \tag{40}$$

which then implies

$$\begin{aligned} \Delta P_{X,t+1} - \Delta P_{Y,t+1} &= (\theta + \frac{1}{\phi})(\Delta P_{X,t} - \Delta P_{Y,t}) - \frac{1}{\phi}(\Delta P_{X,t-1} - \Delta P_{Y,t-1}) \\ &\quad + (\varepsilon_{X,t+1} - \varepsilon_{Y,t+1}) - \theta(\varepsilon_{X,t} - \varepsilon_{Y,t}). \end{aligned} \tag{41}$$

Using standard theory (see Hamilton 1994), $\Delta P_{X,t} - \Delta P_{Y,t}$ will be a stable process so long as the roots of

$$\lambda^2 - \lambda(\theta + \frac{1}{\phi}) + \frac{1}{\phi} = 0$$

are all less than one in absolute magnitude. Within the range $\theta > 0$, $\phi > 0$, this will be true so long as

$$0 < \theta < 1, \; \phi > 1.$$

Proof of Propositions 4 and 5: We use the notation

$$\begin{aligned} \widehat{\Gamma}_k &= \text{cov}(\Delta P_{X,t} - \Delta P_{Y,t}, \Delta P_{X,t+k} - \Delta P_{Y,t+k}), \; k \geq 0 \\ \Gamma_k &= \text{cov}(\Delta P_{X,t}, \Delta P_{X,t+k}), \; k \geq 0 \\ \gamma_k &= \text{corr}(\Delta P_{X,t}, \Delta P_{X,t+k}), \; k \geq 0. \end{aligned}$$

Note that since

$$\text{cov}(\Delta P_{X,t}, \Delta P_{X,t+k}) = -\text{cov}(\Delta P_{X,t}, \Delta P_{Y,t+k}), \; k \geq 1, \tag{42}$$

it follows that

$$\widehat{\Gamma}_k = 4\Gamma_k, \; k \geq 1.$$

To prove the first part of the proposition, it suffices to show that $\gamma_1 > 0$ and that $\gamma_{K+1} < 0$ for some $K \geq 1$. Part (ii) of the proposition then follows immediately from equation (42).

First, we show that $\gamma_1 > 0$. Computing the covariance of (41) with $\Delta P_{X,t+1} - \Delta P_{Y,t+1}$, $\Delta P_{X,t} - \Delta P_{Y,t}$, and $\Delta P_{X,t-1} - \Delta P_{Y,t-1}$ in turn, gives

$$\widehat{\Gamma}_0 = (\theta + \frac{1}{\phi})\widehat{\Gamma}_1 - \frac{1}{\phi}\widehat{\Gamma}_2 + (1 - \frac{\theta}{\phi})(2\psi_s^2 + k) \tag{43}$$

$$\widehat{\Gamma}_1(1 + \frac{1}{\phi}) = (\theta + \frac{1}{\phi})\widehat{\Gamma}_0 - \theta(2\psi_s^2 + k) \tag{44}$$

$$\widehat{\Gamma}_2 = (\theta + \frac{1}{\phi})\widehat{\Gamma}_1 - \frac{1}{\phi}\widehat{\Gamma}_0, \tag{45}$$

where

$$k = \frac{2}{n}(1 - \psi_S^2 - \psi_M^2).$$

This gives us three equations in three unknowns, and after some algebra, we obtain

$$\widehat{\Gamma}_1 = \frac{(2\psi_S^2 + k)(1 + \theta)}{(\phi - 1)(1 + \theta + \frac{2}{\phi})} \tag{46}$$

which is positive under the restrictions $0 < \theta < 1$ and $\phi > 1$ that we derived in the preceeding lemma. Therefore, $\gamma_1 > 0$.

To show that $\gamma_{K+1} < 0$ for some $K \geq 1$, it is sufficient to show that

$$\pi = \widehat{\Gamma}_2 + \theta\widehat{\Gamma}_3 + \theta^2\widehat{\Gamma}_4 + \ldots < 0.$$

Taking the covariance of (40) with $\Delta P_{X,t+1} - \Delta P_{Y,t+1}$, we obtain

$$\widehat{\Gamma}_0 = 2\psi_S^2 + k + \frac{\widehat{\Gamma}_1}{\phi} - \frac{1-\theta}{\phi}\pi.$$

Substituting in $\widehat{\Gamma}_1$ from (46) and the implied reduced form for $\widehat{\Gamma}_0$ from (44) gives

$$\begin{aligned} \pi &= -(\frac{\phi}{1-\theta})(\widehat{\Gamma}_0 - (2\psi_S^2 + k) - \frac{\widehat{\Gamma}_1}{\phi}) \\ &= \frac{-(2\psi_S^2 + k)}{(\phi - 1)(1 + \theta + \frac{2}{\phi})} \end{aligned}$$

which is indeed negative under the restrictions $0 < \theta < 1$ and $\phi > 1$ derived in the lemma. This concludes the proof of Proposition 4. The proof of Proposition 5 is identical in structure.

Proof of Proposition 6: Define

$$\Lambda_k = \text{cov}(\Delta P_{i,t}, \Delta P_{i,t+k}).$$

Then using (37), it is simple to show that

$$\begin{aligned} \text{cov}(\Delta P_{i,t}, \Delta P_{j,t+k}) &= \Lambda_k, \text{ for all } i, j \text{ in the same style} \\ \text{cov}(\Delta P_{i,t}, \Delta P_{j,t+k}) &= -\Lambda_k, \text{ for all } i, j \text{ in different styles} \end{aligned}$$

and

$$\Lambda_k = \Gamma_k,\ k \geq 1.$$

The expected return of the individual stock-level momentum strategy is given by

$$\begin{aligned}
E(\sum_{i=1}^{2n} N_{i,t}\Delta P_{i,t+1}) &= \frac{1}{2n} E \sum_{i=1}^{2n} (\Delta P_{i,t} - \Delta P_{M,t})\Delta P_{i,t+1} \\
&= \frac{1}{2n} \sum_{i=1}^{2n} E(\Delta P_{i,t}\Delta P_{i,t+1}) - \frac{1}{4n^2} \sum_{i,j=1}^{2n} E(\Delta P_{i,t}\Delta P_{j,t+1}) \\
&= \frac{1}{2n} \sum_{i=1}^{2n} \text{cov}(\Delta P_{i,t}\Delta P_{i,t+1}) - \frac{1}{4n^2} \sum_{i,j=1}^{2n} \text{cov}(\Delta P_{i,t}\Delta P_{j,t+1}) + \frac{1}{2n} \sum_{i=1}^{2n} (\mu_i - \mu_M)^2 \\
&= \Lambda_1 + 0 + \frac{1}{2n} \sum_{i=1}^{2n} (\mu_i - \mu_M)^2 = \Gamma_1
\end{aligned}$$

since in our simple economy, all stocks have the same mean return μ_i. In the proof of Proposition 4, we showed that $\Gamma_1 > 0$, which means that the expected return is indeed positive.

The expected return of the individual stock-level value strategy is given by

$$\begin{aligned}
E(\sum_{i=1}^{2n} N_{i,t}\Delta P_{i,t+1}) &= \frac{1}{2n} E \left(\sum_{i=1}^{n} (-\frac{1}{\phi} \sum_{k=1}^{t-1} \theta^{k-1} \frac{\Delta P_{X,t-k} - \Delta P_{Y,t-k}}{2}) \Delta P_{i,t+1} \right) \\
&\quad - \frac{1}{2n} E \left(\sum_{i=n+1}^{2n} (-\frac{1}{\phi} \sum_{k=1}^{t-1} \theta^{k-1} \frac{\Delta P_{X,t-k} - \Delta P_{Y,t-k}}{2}) \Delta P_{i,t+1} \right) \\
&= \frac{1}{2} E[(-\frac{1}{\phi} \sum_{k=1}^{t-1} \theta^{k-1} \frac{\Delta P_{X,t-k} - \Delta P_{Y,t-k}}{2})(\Delta P_{X,t+1} - \Delta P_{Y,t+1})] \\
&= -\frac{1}{4\phi}(\widehat{\Gamma}_2 + \theta\widehat{\Gamma}_3 + \theta^2\widehat{\Gamma}_4 + \ldots) - \frac{1-\theta}{4\phi}(\mu_X - \mu_Y)^2 \\
&= -\frac{\pi}{4\phi}
\end{aligned}$$

since all securities have the same expected return in our economy, and where π is defined in the proof of Proposition 4. In that proof, we showed that $\pi < 0$, which implies that the expected return of the individual stock-level value strategy is indeed positive.

Proof of Proposition 7, part (i): This part of the proposition is trivially true for value strategies since the share demands of a style-level value strategy are identical to the share demands of an individual stock-level value strategy.

We now show that the Sharpe ratio of a style-level momentum strategy is strictly greater than that of the individual stock-level momentum strategy. We do this by showing that both strategies have the same expected return, but that the style-level strategy has a lower expected *squared* return, and hence a lower variance.

The expected return of a style-level momentum strategy is

$$\begin{aligned}
& E\left(\sum_{i=1}^{n}\frac{1}{2n}(\frac{\Delta P_{X,t}-\Delta P_{Y,t}}{2})\Delta P_{i,t+1}+\sum_{i=n+1}^{2n}\frac{1}{2n}(\frac{\Delta P_{Y,t}-\Delta P_{X,t}}{2})\Delta P_{i,t+1}\right) \\
= & \frac{1}{4}E[(\Delta P_{X,t}-\Delta P_{Y,t})(\Delta P_{X,t+1}-\Delta P_{Y,t+1})] \\
= & \Gamma_1+\frac{1}{4}(\mu_X-\mu_Y)^2=\Gamma_1.
\end{aligned}$$

This is indeed equal to the expected return of the individual stock-level momentum strategy, computed in the proof of Proposition 6.

The expected *squared* return of a style-level momentum strategy is given by

$$\begin{aligned}
& \frac{1}{16}E[(\Delta P_{X,t}-\Delta P_{Y,t})^2(\Delta P_{X,t+1}-\Delta P_{Y,t+1})^2] \\
= & E[(\Delta P_{X,t}-\Delta P_{M,t})^2(\Delta P_{X,t+1}-\Delta P_{M,t+1})^2] \\
= & \mathrm{cov}[(\Delta P_{X,t}-\Delta P_{M,t})^2,(\Delta P_{X,t+1}-\Delta P_{M,t+1})^2] \\
& +E(\Delta P_{X,t}-\Delta P_{M,t})^2E(\Delta P_{X,t+1}-\Delta P_{M,t+1})^2.
\end{aligned} \tag{47}$$

Substituting in

$$\Delta P_{X,t+1}-\Delta P_{M,t+1}=\varepsilon_{X,t+1}-\varepsilon_{M,t+1}+\frac{\Delta N^S_{X,t+1}}{\phi},$$

and taking the expectation, (47) eventually reduces to

$$\begin{aligned}
& \mathrm{cov}(\frac{\Delta N^2_{X,t+1}}{\phi^2},(\varepsilon_{X,t}-\varepsilon_{M,t})^2+2(\varepsilon_{X,t}-\varepsilon_{M,t})\frac{\Delta N_{X,t}}{\phi}+\frac{\Delta N^2_{X,t}}{\phi^2}) \\
& +(\frac{\psi_S^2}{2}+\frac{1}{2n}(1-\psi_M^2-\psi_S^2)+\frac{\mathrm{var}(\Delta N_{X,t})}{\phi^2})^2.
\end{aligned} \tag{48}$$

The expected squared return of the individual stock-level momentum strategy is given by

$$\begin{aligned}
& \frac{1}{4n^2}E\left(\sum_{i,j=1}^{2n}(\Delta P_{i,t}-\Delta P_{M,t})(\Delta P_{j,t}-\Delta P_{M,t})\Delta P_{i,t+1}\Delta P_{j,t+1}\right) \\
= & \frac{1}{4n^2}\sum_{i,j=1}^{2n}\mathrm{cov}[(\Delta P_{i,t}-\Delta P_{M,t})(\Delta P_{j,t}-\Delta P_{M,t}),\Delta P_{i,t+1}\Delta P_{j,t+1}] \\
& +\frac{1}{4n^2}\sum_{i,j=1}^{2n}E[(\Delta P_{i,t}-\Delta P_{M,t})(\Delta P_{j,t}-\Delta P_{M,t})]E[\Delta P_{i,t+1}\Delta P_{j,t+1}].
\end{aligned}$$

Substituting in the expressions for $\Delta P_{i,t} - \Delta P_{M,t}$ and $\Delta P_{i,t+1}$ from equations (37) and (38), the expected squared return reduces to

$$\begin{aligned} & \operatorname{cov}(\frac{\Delta N^2_{X,t+1}}{\phi^2}, (\varepsilon_{X,t} - \varepsilon_{M,t})^2 + 2(\varepsilon_{X,t} - \varepsilon_{M,t})\frac{\Delta N_{X,t}}{\phi} + \frac{\Delta N^2_{X,t}}{\phi^2}) \\ & +(\frac{\psi_S^2}{2} + \frac{\operatorname{var}(\Delta N_{X,t})}{\phi^2}) \\ & +\frac{1 - \psi_M^2 - \psi_S^2}{4n^2}(\frac{4n \operatorname{var}(\Delta N_{X,t})}{\phi^2} + (2n-1)(1 - \psi_M^2 - \psi_S^2) + 2n\psi_S^2). \end{aligned} \tag{49}$$

It is now simple to show that (49) is strictly greater than (48). The style level momentum strategy does indeed have the lower variance and hence the higher Sharpe ratio.

Proof of Proposition 7, part (ii): This part of the proposition is trivially true for value strategies since the share demands of a within-style value strategy are identically zero. The expected return of the within-style momentum strategy is

$$\begin{aligned} & \frac{1}{n} E\left(\sum_{i=1}^{n}(\Delta P_{i,t} - \Delta P_{X,t})\Delta P_{i,t+1}) + \sum_{i=n+1}^{2n}(\Delta P_{i,t} - \Delta P_{Y,t})\Delta P_{i,t+1}\right) \\ = & \sum_{i=1}^{n}(\mu_i - \mu_X)^2 + \sum_{i=n+1}^{2n}(\mu_i - \mu_Y)^2 = 0 \end{aligned}$$

in our economy, since all stocks have the same expected return.

Proof of Proposition 8: We restrict the arbitrageur to style level strategies in which demand for all assets within the same style is identical:

$$\begin{aligned} N_{i,t} &= \frac{1}{n} N_{X,t},\ i \epsilon X, \\ N_{j,t} &= \frac{1}{n} N_{Y,t},\ j \epsilon Y. \end{aligned}$$

Given capital of W^A at time t, he solves[25]

$$\begin{aligned} & \max_{N_{i,t}, N_{j,t}} E_t^A(-\exp[-\gamma(W^A + \sum_{i \epsilon X} N_{i,t}\Delta P_{i,t+1} + \sum_{j \epsilon Y} N_{j,t}\Delta P_{j,t+1})]) \\ = & \max_{N_{X,t}, N_{Y,t}} E_t^A(-\exp[-\gamma(W^A + N_{X,t}\Delta P_{X,t+1} + N_{Y,t}\Delta P_{Y,t+1})]), \end{aligned}$$

to obtain

$$\begin{pmatrix} N^A_{X,t} \\ N^A_{Y,t} \end{pmatrix} = V_t^{-1} \begin{pmatrix} \frac{1}{\gamma} E_t^A(P_{X,t+1} - P_{X,t}) \\ \frac{1}{\gamma} E_t^A(P_{Y,t+1} - P_{Y,t}) \end{pmatrix}, \tag{50}$$

where V_t is the conditional covariance matrix of price changes.

[25]The "A" superscript in these expressions stands for arbitrageur.

Since he knows that prices are determined by (17), he is able to conclude that

$$\begin{aligned} E_t^A(P_{X,t+1} - P_{X,t}) &= \frac{1}{\phi}\Delta N_{X,t+1}^S, \\ E_t^A(P_{Y,t+1} - P_{Y,t}) &= -\frac{1}{\phi}\Delta N_{X,t+1}^S. \end{aligned} \tag{51}$$

Equation (50) then reduces to

$$\begin{aligned} N_{X,t}^A &= c\Delta N_{X,t+1}^S, \\ N_{Y,t}^A &= -c\Delta N_{X,t+1}^S, \end{aligned} \tag{52}$$

where c is a positive constant that depends on the covariance matrix of returns.

Proof of Proposition 9: This proposition follows directly from Propositions 2, 3, and 4, with X taken to be the index I, and Y taken to be the set of stocks outside the index, NI.

Table 1: Own- and cross-autocorrelations of raw and market-adjusted returns on styles X and Y in an economy with switchers.

	$\text{corr}(\Delta P_{X,t}, \Delta P_{X,t-k})$	$\text{corr}(\Delta P_{X,t}, \Delta P_{Y,t-k})$
$k = 1$	0.50	-0.50
$k = 2$	0.36	-0.36
$k = 3$	0.23	-0.23
$k = 4$	0.12	-0.12
$k = 5$	0.02	-0.02
$k = 6$	-0.06	0.06
$k = 7$	-0.11	0.11
$k = 8$	-0.15	0.15
$k = 9$	-0.18	0.18
	$\text{corr}(\Delta P_{X,t} - \Delta P_{M,t}, \Delta P_{X,t-k} - \Delta P_{M,t-k})$	$\text{corr}(\Delta P_{X,t} - \Delta P_{M,t}, \Delta P_{Y,t-k} - \Delta P_{M,t-k})$
$k = 1$	0.78	-0.78
$k = 2$	0.56	-0.56
$k = 3$	0.36	-0.36
$k = 4$	0.18	-0.18
$k = 5$	0.02	-0.02
$k = 6$	-0.10	0.10
$k = 7$	-0.19	0.19
$k = 8$	-0.24	0.24
$k = 9$	-0.28	0.28

Table 2: Sharpe ratios of various strategies in an economy with switchers. An individual stock-level momentum strategy buys stocks that performed better than the average stock last period; a style-level momentum strategy buys styles that did better than the average style last period; and a within-style momentum strategy buys stocks that did better than their style last period. An individual stock-level value strategy buys stocks trading below fundamental value; a style-level value strategy buys styles trading below fundamental value; and a within-style value strategy buys stocks trading at a bigger discount to fundamental value than their style. The optimal strategy is the one that would be chosen by an arbitrageur who knows the correct process followed by prices in the economy.

Strategy	Sharpe Ratio
Momentum	
Individual stock-level	0.61
Style-level	0.61
Within-style	0
Value	
Individual stock-level	0.12
Style-level	0.12
Within-style	0
Optimal	0.62

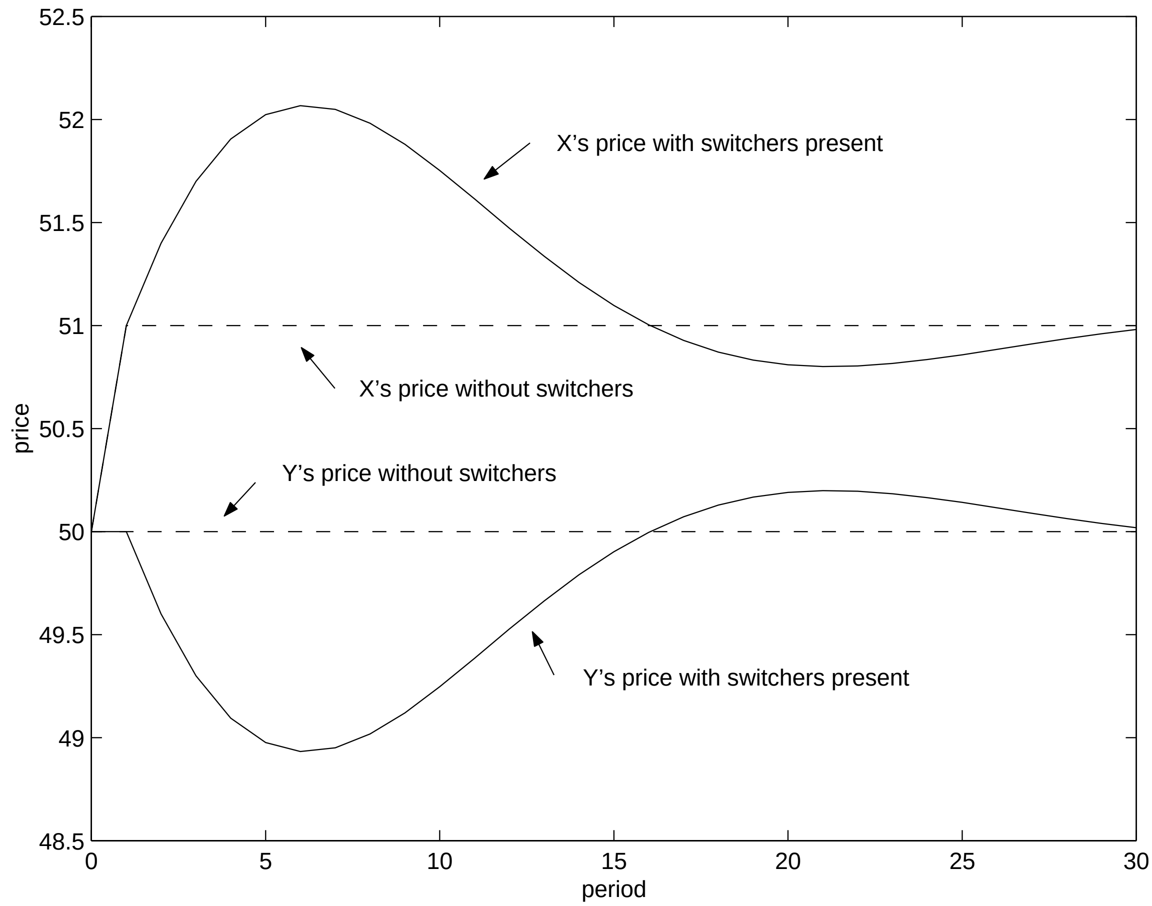

Figure 1. Impulse responses to a cash-flow shock to style X in period 1. Prices of both X and Y are initially 50. Dashed lines indicate fundamental values, or prices without switchers. Solid lines indicate prices with switchers.

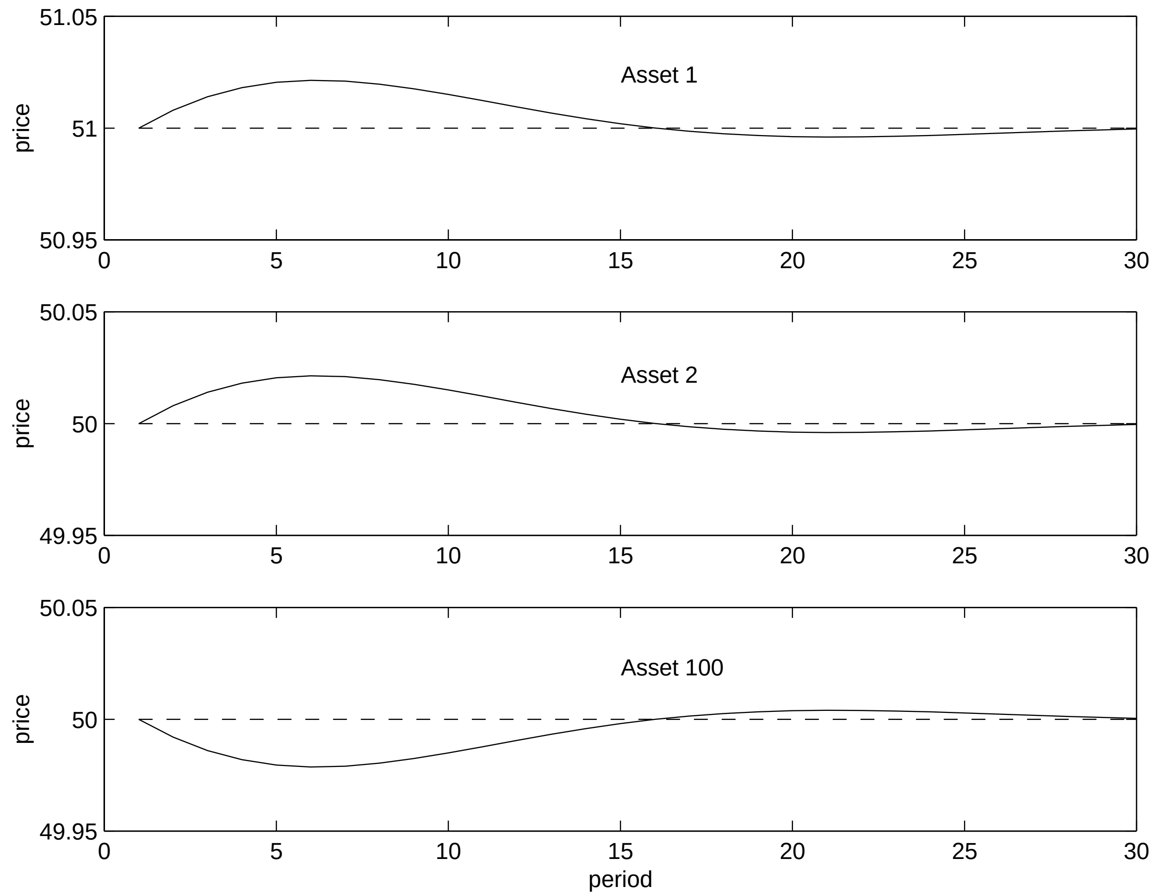

Figure 2. The graph shows the prices (solid lines) and fundamental values (dashed lines) of assets 1, 2 and 100 after a cash-flow shock to asset 1 at time 1. All assets have an initial price of 50 at time 0. Assets 1 and 2 are in style X while asset 100 is in style Y.

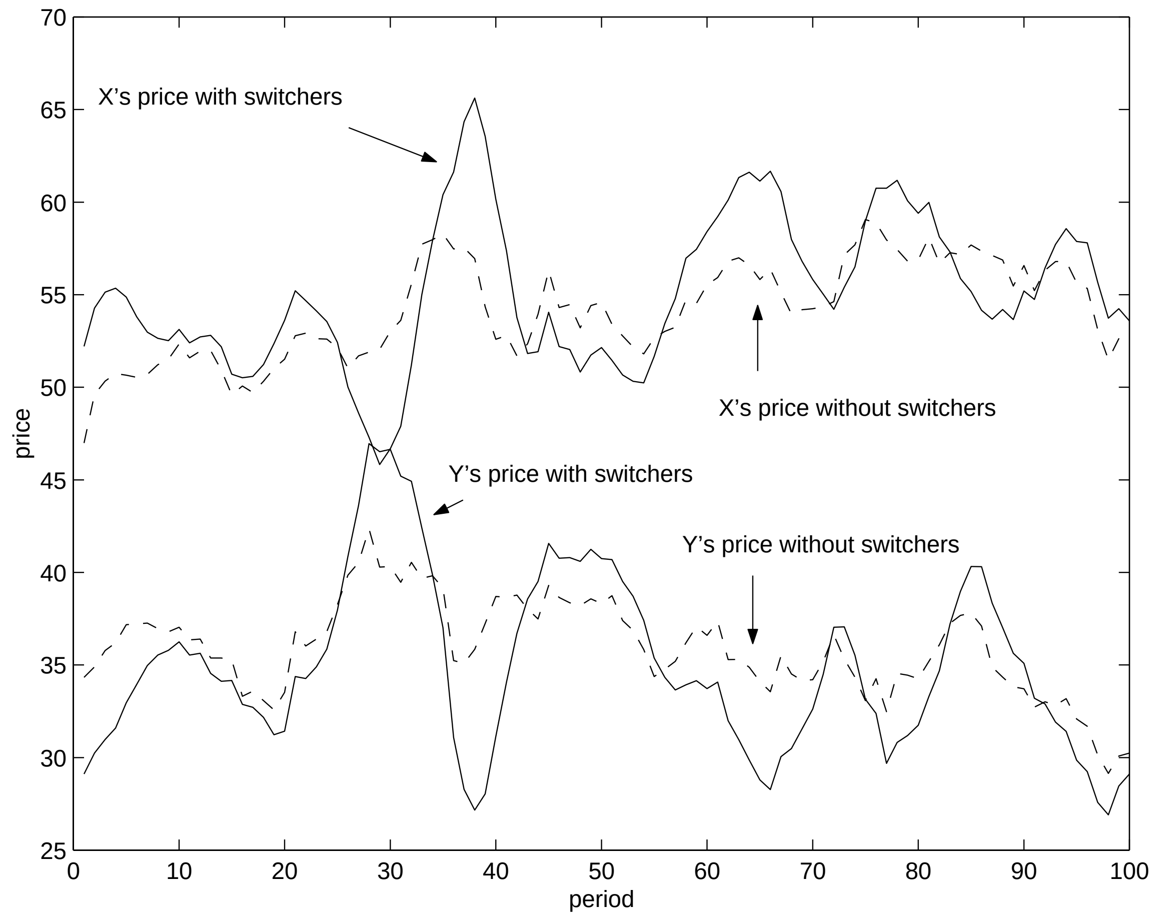

Figure 3. Price paths for two styles X and Y. Dashed lines indicate fundamental values, or prices without switchers. Solid lines indicate prices with switchers.